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EXPENSES ANALYSIS FOR A COMPLEX SYSTEM WORKING UNDER DIFFERENT WEATHER CONDITIONS

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ABSTRACT:
In this paper, the author has estimated the availability and expenses function for a complex system which works in different weather conditions. The whole system under consideration consists of two distinct components working in parallel redundancy. On failure of one component the system works in reduced efficiency state. This failed unit can be repaired immediately. When both the units get failed, the whole system state being failed and the system has to wait for repair under these circumstances. When this system works in different weather conditions, the failure and repair rates will be different.

Key words: redundancy, circumstance, failure

INTRODUCTION:
Since, the system is of Non-Markovian nature, the author has introduced some supplementary variables to make this system markovian. Laplace transform has used to solve the mathematical model. All the failure rates follow exponential time distribution whereas all repair rates follow general time distribution. The whole system can also fail due to human error. The mathematical expression for various transition-states probabilities, availability function and profit function for the system has computed. To improve practical utility of the model, steady-state probability for different transition states and a particular case (when repairs follow exponential time distribution) have also obtained. A numerical computation together with its graphical illustration has been appended at last to highlight important results of the study.
The results obtained in the study are of much importance for various systems of practical utility and can be used as it is for the similar configurations. For example, it is the time of computer and we can find several parts of it with similar configuration that ability differs for different weather conditions. If we don't care for these parts, it causes a big loss not only of important data saved in it but also of money and time to remove such problem.
Also we can consider the example of aeroplane as it consists various parts with it of similar configuration and their ability differs with different weather conditions. If we don't care for these it also causes a big loss not only of money but also of many lives that travel in it. Thus, we can use the results obtained in this study to those sensitive parts to prevent a big loss.

ASSUMPTIONS
The following assumptions have been associated with this study:
(i) Initially the whole system is good and working with full efficiency.
(ii) System can be repaired immediately on failure of one unit of the system but it has to wait for repair in case both units get failed.
(iii) All failures and waiting rates follow exponential time distribution whereas all repairs follow general time distribution.
(iv) Repairs are perfect, i.e., the system works like new after repair.
(v) The whole system can also fail due to human initiated errors.

LIST OF NOTATIONS

$\lambda_i(i=1,2--n)$		Failure rate of first unit of the system when it works in i^{th} weather condition.
$\mu_i(i=1,2--n)$		Failure rate of second unit of the system when it works in i^{th} weather condition.
W		Waiting rate to repair both units of system.
H		Human error rate.
$\beta_i(x)\Delta/\gamma_i(y)\Delta$ $(i=1,2,--n)$		The first order probability that one /two units of the system will be repaired in the time interval $(x,x+\Delta)/\ (y,y+\Delta)$ conditioned that it was not repaired up to the time x/y, system is working in i^{th} weather condition.
$\alpha_h(z)\Delta$		The first order probability that the human error will be repaired in the time interval $(z,z+\Delta)$, conditioned that it was not repaired up to the time z.
$P_0(\mathrm{t})$	:	Pr {system is operable at time t}.
$P_i^1(x,t)\Delta$	:	Pr {system is degraded due to failure of one unit at time t}. Elapsed repair time for this failure in i^{th} weather condition, lies in the interval $(x,x+\Delta)$.
$P_i^2(t)$	:	Pr {system is fail at time t due to failure of both units of the system}. It is working in i^{th} weather condition & it has to wait for repair.
$P_i^{2R}(y,t)\Delta$	:	Pr {the system is ready for repair of two units at time t}. Elapsed repair time for this in i^{th} weather condition lies in the interval $(y,y+\Delta)$.
$P_h(z,t)\Delta$	:	Pr {the system is failed due to human error at time t}. Elapsed repair time lies in the interval $(z,z+\Delta)$.

FORMULATION OF MATHEMATICAL MODEL

By using continuity argument and limiting procedure, we obtain the following set of difference-differential equations, which is continuous in time and discrete in space, governing the behavior of considered model:

$$\left[\frac{d}{dt}+\sum_{i=1}^{n}\lambda_i+h\right]P_0(t)=\sum_{i=1}^{n}\int_0^\infty P_i^1(x,t)\beta_i(x)dx+\sum_{i=1}^{n}\int_0^\infty P_i^{2R}(y,t)\gamma_i(y)dy+\int_0^\infty P_h(z,t)\alpha_i(z)dz \qquad \ldots(1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_i+\beta_i(x)\right]P_i^1(x,t)=0,\quad \forall i=1,2,-----n \qquad \ldots(2)$$

$$\left[\frac{d}{dt}+w\right]P_i^2(t)=\int_0^\infty P_i^1(x,t)\mu_i dx,\quad \forall i=1,2,-----n \qquad \ldots(3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\gamma_i(y)\right]P_i^{2R}(y,t)=0,\quad \forall i=1,2,-----n \qquad \ldots(4)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\alpha_h(z)\right]P_h(z,t)=0 \qquad \ldots(5)$$

Boundary conditions are:

$$P_i^1(0,t)=\lambda_i P_0(t), \quad \forall i=1,2,-----n \qquad \dots (6)$$

$$P_i^{2R}(0,t)=wP_i^2(t), \quad \forall i=1,2,-----n \qquad \dots (7)$$

$$P_h(0,t)=hP_0(t) \qquad \dots (8)$$

Initial conditions are:

$$P_0(t)=1, \text{ otherwise zero} \qquad \dots (9)$$

SOLUTION OF THE MODEL

We shall solve the above system of difference-differential equations with the aid of Laplace transform to obtain probabilities of different transition states. Thus, taking Laplace transform of equations (1) through (8) subjected to initial conditions (9), one can obtain:

$$\left[s+\sum_{i=1}^{n}\lambda_i+h\right]\overline{P}_0(s)=1+\sum_{i=1}^{n}\int_0^\infty \overline{P}_i^1(x,s)\beta_i(x)dx+\sum_{i=1}^{n}\int_0^\infty \overline{P}_i^{2R}(y,s)\gamma_i(y)dy \qquad \dots (10)$$

$$+\int_0^\infty \overline{P}_h(z,s)\alpha_h(z)dz$$

$$\left[\frac{\partial}{\partial x}+s+\mu_i+\beta_i(x)\right]\overline{P}_i^1(x,s)=0, \quad \forall i=1,2,-----n \qquad \dots (11)$$

$$[s+w]\overline{P}_i^2(s)=\mu_i\overline{P}_i^1(s), \quad \forall i=1,2,-----n \qquad \dots (12)$$

$$\left[\frac{\partial}{\partial y}+s+\gamma_i(y)\right]\overline{P}_i^{2R}(y,s)=0, \quad \forall i=1,2,-----n \qquad \dots (13)$$

$$\left[\frac{\partial}{\partial z}+s+\alpha_h(z)\right]\overline{P}_h(z,s)=0 \qquad \dots (14)$$

$$\overline{P}_i^1(0,s)=\lambda_i\overline{P}_0(s), \quad \forall i=1,2,-----n \qquad \dots (15)$$

$$\overline{P}_i^{2R}(0,s)=w\overline{P}_i^2(s), \quad \forall i=1,2,-----n \qquad \dots (16)$$

and $\overline{P}_h(0,s)=h\overline{P}_0(s)$ … (17)

Now, integrating equation (11) by using the boundary condition (15), we get; $\forall i=1,2,-----n$:

$$\overline{P}_i^1(x,s)=\overline{P}_i^1(0,s)e^{-(s+\mu_i)x-\int\beta_i(x)dx}$$

$$=\lambda_i\overline{P}_0(s)e^{-(s+\mu_i)x-\int\beta_i(x)dx}$$

$$\Rightarrow \overline{P}_i^1(s)=\lambda_i\overline{P}_0(s)\frac{1-\overline{S}_{\beta_i}(s+\mu_i)}{s+\mu_i}$$

or $\overline{P}_i^1(s)=\lambda_i\overline{P}_0(s)D_{\beta_i}(s+\mu_i)$ say

$$\forall i=1,2,-----n \qquad \dots (18)$$

By making use to equation (18), equation (12) gives on simplification:

$$\overline{P}_i^2(s)=\frac{\lambda_i\mu_i}{(s+w)}\overline{P}_0(s)D_{\beta_i}(s+\mu_i) \qquad \dots (19)$$

$$\forall i=1,2,-----n$$

Next, integrating equation (13) by using boundary condition (16) and the relations (19), we have:

$$\overline{P}_i^{2R}(y,s) = \overline{P}_i^{2R}(0,s)e^{-sy-\int \gamma_i(y)dy}$$
$$= w\overline{P}_i^{2}(s)e^{-sy-\int \gamma_i(y)dy}$$
$$\Rightarrow \overline{P}_i^{2R}(s) = w\overline{P}_i^{2}(s)\frac{1-\overline{S}_{\gamma_i}(s)}{s}$$
$$= w\overline{P}_i^{2}(s)D_{\gamma_i}(s) \text{ say}$$

or, $$\overline{P}_i^{2R}(s) = \frac{\lambda_i\mu_i w}{(s+w)}\overline{P}_0(s)D_{\beta_i}(s+\mu_i)D_{\gamma_i}(s) \quad \ldots (20)$$

$\forall i = 1,2,-----n$

Again equation (14) gives on integration by making use of boundary condition (17):

$$\overline{P}_h(z,s) = \overline{P}_h(0,s)e^{-sz-\int \alpha_h(z)dz}$$
$$= h\overline{P}_0(s)e^{-sz-\int \alpha_h(z)dz}$$
$$\Rightarrow \overline{P}_h(s) = h\overline{P}_0(s)\frac{1-\overline{S}_h(s)}{s}$$

or, $$\overline{P}_h(s) = h\overline{P}_0(s)D_h(s) \text{ say} \quad \ldots (21)$$

Finally, equation (10) gives on simplification by using relevant results:

$$\overline{P}_0(s) = \frac{1}{A(s)}$$

Where $$A(s) = s + \sum_{i=1}^{n}\lambda_i + h - \sum_{i=1}^{n}\lambda_i\overline{S}_{\beta_i}(s+\mu_i) - h\overline{S}_h(s)$$
$$-\sum_{i=1}^{n}\frac{\lambda_i\mu_i w}{(s+w)}D_{\beta_i}(s+\mu_i)\overline{S}_{\gamma_i}(s) \quad \ldots (22)$$

Thus, we have the following Laplace transforms of various transition-states probabilities:

$$\overline{P}_0(s) = \frac{1}{A(s)} \quad \ldots (23)$$

$$\overline{P}_i^{1}(s) = \frac{\lambda_i}{A(s)}D_{\beta_i}(s+\mu_i), \quad \forall i = 1,2,-----n \quad \ldots (24)$$

$$\overline{P}_i^{2}(s) = \frac{\lambda_i\mu_i}{A(s)(s+w)}D_{\beta_i}(s+\mu_i) \quad \forall i = 1,2,-----n \quad \ldots (25)$$

$$\overline{P}_i^{2R}(s) = \frac{\lambda_i\mu_i w}{A(s)(s+w)}D_{\beta_i}(s+\mu_i)D_{\gamma_i}(s) \quad \forall i = 1,2,-----n \quad \ldots (26)$$

$$\overline{P}_h(s) = \frac{h}{A(s)}D_h(s) \quad \ldots (27)$$

Where, $D_i(j) = \frac{i}{i+j}, \forall\, i \text{ and } j$

And $A(s)$ has been given in equation (22).

It is worth noticing that

$$\overline{P}_0(s) + \sum_{i=1}^{n}\left[\overline{P}_i^{1}(s) + \overline{P}_i^{2}(s) + \overline{P}_i^{2R}(s)\right] + \overline{P}_h(s) = \frac{1}{s} \quad \ldots (28)$$

ASYMPTOTIC BEHAVIOR OF THE SYSTEM

Using Abel's Lemma, viz., $\underset{t\to\infty}{Lim}\, F(t) = \underset{s\to 0}{Lim}\, s\overline{F}(s) = F(say)$, provided the limit on L.H.S exists, in equations (23) through (27), we have following steady state probabilities:

$$P_0 = \frac{1}{A'(0)} \qquad \ldots (29)$$

$$P_i^1 = \frac{\lambda_i}{A'(0)} D_{\beta_i}(\mu_i), \quad \forall i = 1,2,-----n \qquad \ldots (30)$$

$$P_i^2 = \frac{\lambda_i \mu_i}{wA'(0)} D_{\beta_i}(\mu_i) \quad \forall i = 1,2,-----n \qquad \ldots (31)$$

$$P_i^{2R} = \frac{\lambda_i \mu_i}{A'(0)} D_{\beta_i}(\mu_i) M_{\gamma_i} \quad \forall i = 1,2,-----n \qquad \ldots (32)$$

$$and \quad P_h = \frac{h}{A'(0)} M_h \qquad \ldots (33)$$

Where, $A'(0) = \left[\frac{d}{ds} A(s)\right]_{s=0}$

And $M_a = -\overline{S}'_a(0) =$ mean time to repair a^{th} component.

PARTICULAR CASE

When all repairs follow exponential time distribution:

In this case, setting $\overline{S}_a(b) = \frac{a}{b+a}$ for all a and b, in equations (23) through (27), we can obtain the following transition – states probabilities:

$$\overline{P}_0(s) = \frac{1}{B(s)} \qquad \ldots (34)$$

$$\overline{P}_i^1(s) = \frac{\lambda_i}{B(s)} \cdot \frac{1}{s + \mu_i + \beta_i}, \quad \forall i = 1,2,-----n \qquad \ldots (35)$$

$$\overline{P}_i^2(s) = \frac{\lambda_i \mu_i}{B(s)(s+w)} \cdot \frac{1}{s + \mu_i + \beta_i} \quad \forall i = 1,2,-----n \qquad \ldots (36)$$

$$\overline{P}_i^{2R}(s) = \frac{\lambda_i \mu_i w}{B(s)(s+w)} \cdot \frac{1}{s + \mu_i + \beta_i} \cdot \frac{1}{s + \gamma_i}, \quad \forall i = 1,2,-----n \qquad \ldots (37)$$

$$\text{and } \overline{P}_h(s) = \frac{h}{B(s)} \cdot \frac{1}{s + \alpha_h} \qquad \ldots (38)$$

Where,

$$B(s) = s + \sum_{i=1}^{n} \lambda_i + h - \sum_{i=1}^{n} \frac{\lambda_i \beta_i}{s + \mu_i + \beta_i} - \frac{h\alpha_h}{s + \alpha_h} - \sum_{i=1}^{n} \frac{\lambda_i \mu_i w \gamma_i}{(s+w)(s + \mu_i + \beta_i)(s + \gamma_i)}$$

AVAILABILITY OF CONSIDERED SYSTEM

We have

$$P_{up}(s) = \overline{P}_0(s) + \sum_{i=1}^{\eta} \overline{P}_i^1(s)$$

Putting the values on R.H.S and on taking inverse Laplace transform, we may obtain the up state probability of the whole system and is given by:

$$P_{up}(t) = (1+E)e^{-(\lambda+h)t} - Ee^{-\mu t} \qquad \dots (40)$$

where, $\sum_{i=1}^{n} \lambda_i = \lambda, \sum_{i=1}^{n} \mu_i = \mu$

and $E = \dfrac{\lambda}{\mu - \lambda - h}$ … (41)

Also, the down state probability is

$$P_{down}(t) = 1 - P_{up}(t) \qquad \dots (42)$$

It is interesting to note here that $P_{up}(0) = 1$

EXPENSES FUNCTION FOR THE CONSIDERED SYSTEM

Expenses function for the system can be obtained by the formula.

$$G(t) = C_1 \int_0^t P_{up}(t)dt - C_2 t \qquad \dots (43)$$

where, C_1 is the revenue per unit time and C_2 is the repair expenses per unit time. Using (40), equation (43) gives:

$$G(t) = C_1 \left\{ \frac{(1+E)}{(\lambda+h)} \left[1 - e^{-(\lambda+h)t}\right] - \frac{E}{\mu}\left[1 - e^{-\mu t}\right] \right\} - C_2 t \qquad \dots (44)$$

where, E, λ and μ are explained earlier.

NUMERICAL ILLUSTRATION

To observe the variations in the values of availability and profit function w.r.t. the time'‘t’, let us consider the following numerical computation:

$\lambda = 0.001, \mu = 0.002, h = 0.02, C_1 = Rs.5.00$/ Unit time, $C_2 = \text{Re}.1.00$/ unit time and t=0,1,2----------.

Using these values in equation (40), one can compute the table-1 and the corresponding graph has shown in fig-1. By using this numerical illustration in equation (44), we compute the table -2 and the corresponding graph has shown through fig-2.

Table-1

t	$P_{up}(t)$
0	1
1	0.980208
2	0.960824
3	0.941842
4	0.923252
5	0.905047
6	0.887218
7	0.869757
8	0.852657
9	0.835911
10	0.819511

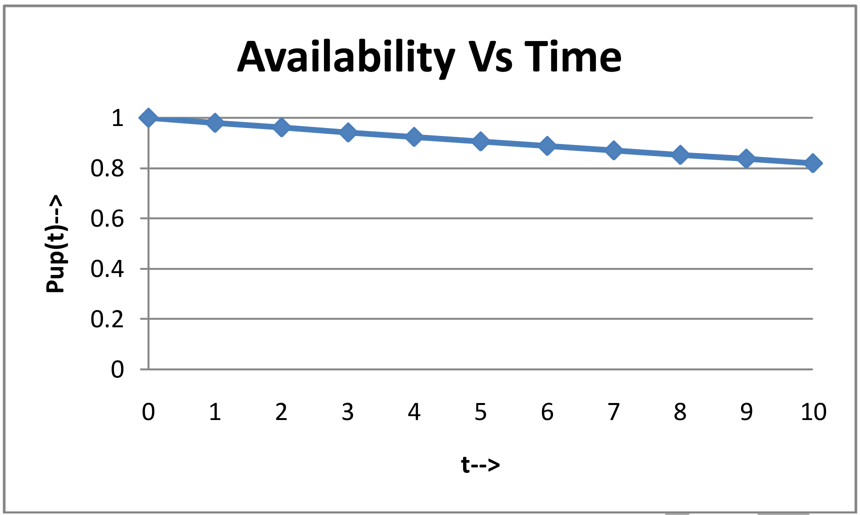

Fig- 1

Table-2

t	G(t)
0	0
1	3.950347
2	7.802758
3	11.55926
4	15.22183
5	18.79242
6	22.27293
7	25.66521
8	28.9711
9	32.19238
10	35.33079

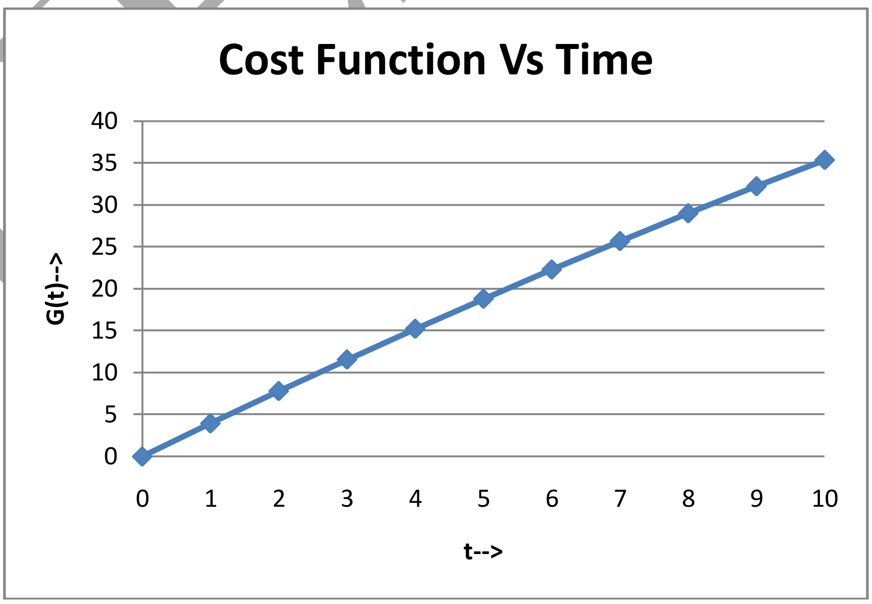

Fig-2

RESULTS AND DISCUSSION:

In this paper, the researcher has computed some important reliability parameters for a complex system which is working in different weather conditions. Supplementary variables technique and Laplace transform have used to formulate and solve the mathematical model. Asymptotic behavior of the system, a particular case has also appended to improve practical utility of the model. A numerical computation has considered highlighting important results.

Table-1 and fig-1 reveal that the availability of the system is decreasing smoothly and in uniform manner as we make increase in time t. A critical examination of table-2 and fig-2 yields that expenses function of considered system increases approximately in constant manner with time t.

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