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IMMOVABILITY OF STRATIFIED FLOWS OF MICROPOLAR FLUIDS IN POROUS MEDIUM

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ABSTRACT

Fluid mechanics can be divided into fluid statics, the study of fluids at rest; fluid kinematics, the study of fluids in motion; and fluid dynamics, the study of the effect of forces on fluid motion. Fluid mechanics concerns itself with the investigation of motion and equilibrium of fluids. We normally recognize three states of matter: solid, liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it flows under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely.

KEY WORDS: Fluids, deformation

INTRODUCTION

Micropolar fluids are fluids with microstructure belonging to a class of non-Newtonian fluids with non-symmetrical stress tensor. Micropolar fluids have the micro-rotational effects and micro-inertia effects. The concept of micro-rotation was initially proposed by Cosserat and Cosserat (2016) which was applied successfully to describe the flow of fluids with micro-structures [Condiff and Dahler (2012). Inspired by them, Eringen (1964) analyzed a new class of fluids called "micro fluids" exhibiting micro effects similar to simple micro-elastic materials. In these fluids local structures and micro-rotations of the material particles contained in each of its volume element i.e. gyration effects play an important role. The stresses and stress moments are functions of deformation rate tensors and various micro-deformation rate tensors and hence these types of fluids are quite complicated to the extent that even in the simplest case of constitutive linear theory, these contain 22 viscosity coefficients. Therefore it is not easily amenable to construct and analyze the mathematical models of such type of problems. Eringen (1966) introduced a subclass of these fluids called micropolar fluids, in which micro-rotational effects like micro-rotational inertia are important and included in the analysis but micro stress stretch of the particles is not allowed. These fluids can support the couple stress, the body couples, the non-symmetric stress tensor and a rotation field independent of velocity field. The theory, thus, has two independent kinetics variables: the velocity vector and the spin or micro-rotation velocity vector. This theory involves only four additional viscosity coefficients, one introduced through the linear constitutive equation for a non-symmetric stress tensor and other three due to linear constitutive equation for a couple stress, hence converts to a simple, elegant and realistic theory.

REVIEW OF LITERATURE

The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood (2005) and Wooding (2010). Agarwal and Dhanpal (1988) examined the classical couette-poiseuille flow and heat transfer in micropolar fluids between two co-axial porous circular cylinders. Desseaux and Kelson (2010) considered boundary layer flow of a micropolar fluid driven by a porous stretching sheet. Kelson et al. (2013) investigated two dimensional flow of a micropolar fluid driven by suction or injection in a porous channel. Zakaria (2014) investigated the influence of a transverse magnetic field on the motion of an electrically conducting micropolar fluid through a porous medium in one-dimension. Kamal et al. (2006) analyzed numerically the steady viscous flow of a micropolar fluid driven by injection between two porous disks. Naduvinamani and Marali (2008) studied the rheological effects of micropolar fluid lubricants on the steady state and dynamic behavior of porous slider bearings by considering the squeezing action. Rahman (2009) numerically investigated the steady laminar free-forced convective flow and heat transfer of micropolar fluids past a vertical radiate isothermal permeable surface with viscous dissipation and Joule heating. Reddy et al. (2010) investigated the oscillatory two-dimensional laminar flow of a viscous incompressible electrically conducting micropolar fluid past a semi-infinite vertical moving porous plate embedded in a porous medium and subjected to a uniform transverse magnetic field in the presence of thermal radiation effects. Islam et al. (2011) numerically examined the MHD micropolar fluid flow through a vertical porous plate using Finite Difference Technique.

FORMULATION OF THE PROBLEM

Consider the stability of an incompressible micropolar fluid, saturated in a porous medium, confined between infinite horizontal free planes at a finite gap d. In the Cartesian frame of reference, the axis of x is in the main flow direction and the axis of z is perpendicular to the planes so that the gravity acts in the negative z-direction.

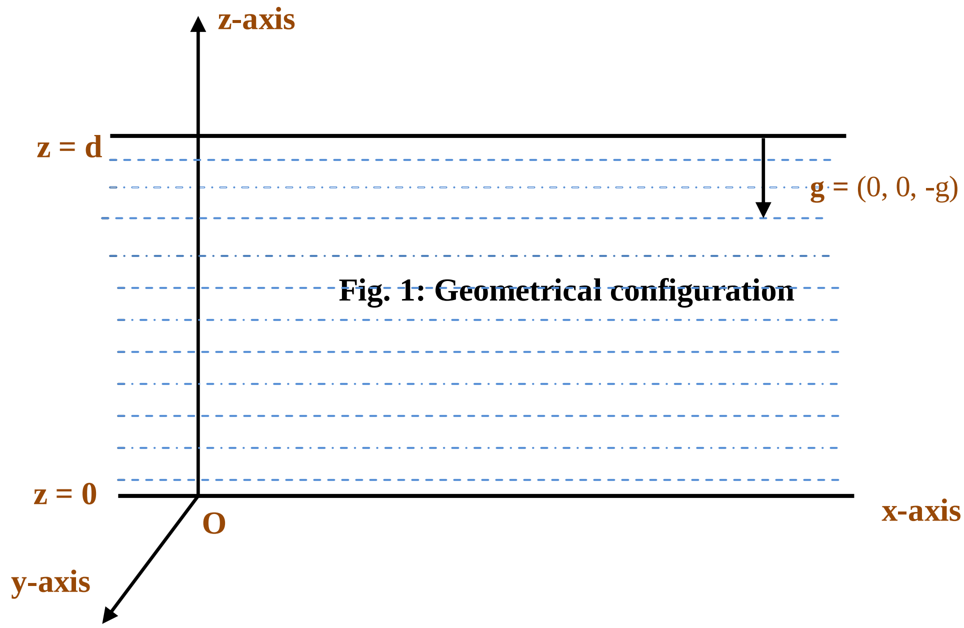

Fig. 1: Geometrical configuration

Let $\mathbf{q}$, $\boldsymbol{\omega}$, p, g, $\mathbf{e}_z$, j, ε and k_1 denote velocity, micro rotation velocity, density, pressure, acceleration due to gravity, unit vector in z-direction, microinertia constant, coefficient of viscosity, porosity and permeability of the porous medium respectively. The parameters ε', β'', γ and μ_r stand for the micropolar coefficients of viscosity.

Obeying Darcy's law the equations governing the flow of micropolar fluid are

$$\nabla.\mathbf{q} = 0, \quad (1.1)$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right] = -\nabla p - \frac{1}{k_1}(\mu + \mu_r)\mathbf{q} + \mu_r(\nabla \times \boldsymbol{\omega}) - \rho g \mathbf{e}_z, \quad (1.2)$$

$$\rho j\left[\frac{\partial \boldsymbol{\omega}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\boldsymbol{\omega}\right] = (\varepsilon' + \beta'')\nabla(\nabla.\boldsymbol{\omega}) + \gamma\nabla^2\boldsymbol{\omega} + \frac{\mu_r}{\varepsilon}(\nabla \times \mathbf{q}) - 2\mu_r\boldsymbol{\omega} \quad (1.3)$$

and

$$\frac{D\rho}{Dt} = 0. \quad (1.4)$$

BASIC STATE

The initial state is such that the fluid is at rest, micropolar velocity $\boldsymbol{\omega}=0$ and the density and pressure depend upon z only. Thus the initial state is characterized by

$$\mathbf{q}=(0,0,0),\quad \boldsymbol{\omega}=(0,0,0),\quad p=p(z) \tag{1.5}$$

NORMAL MODE ANALYSIS

The perturbations are decomposed into wave like components as

$$f'(x,y,z,t)=f(z)\exp\left(ik_x x+ik_y y+\sigma t\right), \tag{1.6}$$

where k_x and k_y are real wave numbers in x and y directions, $k^2=(k_x^2+k_y^2)$ and $\sigma=(\sigma_r+i\sigma_i)$ is the complex number.

On using the non- dimensional parameters

$$(k^*,k_x^*,D^*)=d(k,k_x,D),\quad j^*=\frac{j}{d^2}\ \text{and}\ \sigma^*=\frac{\rho_o\sigma d^2}{\mu},$$

where d is the characteristic length, the final stability governing equation, after dropping stars, is given by

$$\sigma\left(D^2-k^2\right)\left\{\sigma l-\left(D^2-k^2\right)+2A\right\}w+AK\left(D^2-k^2\right)^2w$$
$$+\frac{(1+K)\varepsilon}{k_1}\left\{\sigma l-\left(D^2-k^2\right)+2A\right\}\left(D^2-k^2\right)w$$
$$-\frac{J_\mu k^2}{\sigma}\left\{\sigma l-\left(D^2-k^2\right)+2A\right\}w=0, \tag{1.7}$$

where $l=\frac{j^*A}{K}$, $C_0'=\frac{\gamma}{\mu d^2}$, $K=\frac{\mu_r}{\mu}$, $A=\frac{K}{C_0'}$, $J_\mu=\frac{\rho_o{}^2 d^3 g\beta}{\mu^2}$ and $\beta=-\frac{D\rho_o}{\rho_o}$.

The boundary conditions appropriate for the problem are

$w=0$, $D^2w=0$, at $z=0$ and $z=1$. (1.8)

ANALYTICAL DISCUSSION

Multiply equation (1.9) by w*, the complex conjugate of w, integrate over the range of z and make use of boundary conditions (1.10). The real and imaginary parts of the resulting equation respectively yield

$$\int\left\{\left(\sigma_r^2-\sigma_i^2\right)l+2A\sigma_r+\frac{(1+K)\varepsilon}{k_1}\left(l\sigma_r+2A\right)+\frac{J_\mu k^2\sigma_r}{\sigma_r^2+\sigma_i^2}\right\}\left(|Dw|^2+k^2|w|^2\right)dz$$
$$+\int\left\{\sigma_r-AK+\frac{\varepsilon(1+K)}{k_1}\right\}\left(|D^2w|^2+2k^2|Dw|^2+k^4|w|^2\right)dz \tag{1.9}$$
$$+\int J_\mu k^2\left(\frac{2A\sigma_r}{\sigma_r^2+\sigma_i^2}+l\right)|w|^2dz=0$$

In the analysis given below, two cases have been discussed depending upon whether the system is statically stable.

NUMERICAL DISCUSSION

CASE I: $J_\mu>0$ (i.e. $D\rho_o<0$)

In **Table - 1** values of critical wave number k_c representing the change of oscillatory stable region to oscillatory unstable region for fixed J_μ are given.

Table – 1: Value of critical wave number representing change of oscillatory stable modes into oscillatory unstable modes for variable permeability.

J = 0.25						
k_1	K=1		K=3		K=5	
	k_c oscillatory stable modes	k_c oscillatory unstable modes	k_c oscillatory stable modes	k_c oscillatory unstable modes	k_c oscillatory stable modes	k_c oscillatory unstable modes
2			1.4564	1.4565	1.2616	1.2617
2.1	4.4978	4.4979	1.3613	1.3614	1.1916	1.1917
3	1.4641	1.4642	0.9373	0.9374	0.853	0.8531
4	1.0517	1.0518	0.7487	0.7488	0.6904	0.6905
5	0.8672	0.8673	0.6432	0.6433	0.5967	0.5968
6	0.7562	0.7563	0.5733	0.5734	0.5338	0.5339
7	0.6799	0.68	0.5225	0.5226	0.4877	0.4878
8	0.6232	0.6233	0.4834	0.4835	0.4521	0.4522
9	0.5788	0.5789	0.4520	0.4521	0.4234	0.4235
10	0.5429	0.543	0.4262	0.4263	0.3996	0.3997
11	0.5130	0.5131	0.4044	0.4045	0.3794	0.3795
12	0.4876	0.4877	0.3856	0.3857	0.3622	0.3623
13	0.4657	0.4658	0.3693	0.3694	0.3471	0.3472
14	0.4466	0.4467	0.3549	0.355	0.3338	0.3339
15	0.4296	0.4297	0.3421	0.3422	0.3219	0.322
20	0.3668	0.3669	0.2940	0.2941	0.2771	0.2772
25	0.3254	0.3255	0.2619	0.262	0.2471	0.2472
30	0.2955	0.2956	0.2384	0.2385	0.2252	0.2253
40	0.2543	0.2544	0.2058	0.2059	0.1946	0.1947
50	0.2266	0.2267	0.1837	0.1838	0.1738	0.1739
100	0.1591	0.1592	0.1295	0.1296	0.1226	0.1227

It is clear that as permeability k_1 and micropolar parameter K increase, region for oscillatory unstable increases. Values of critical wave number k_c representing the change from oscillatory unstable region to oscillatory stable region for fixed $k_1 = 3$.

Table – 2: Value of critical wave number representing change of oscillatory stable modes into oscillatory unstable modes for variable J_1.

k_1 = 3						
J_μ	K=1		K=3		K=5	
	k_c oscillatory unstable modes	k_c oscillator y stable modes	k_c oscillatory unstable modes	k_c oscillatory stable modes	k_c oscillatory unstable modes	k_c oscillatory stable modes
0.1	1.434	1.435	0.9121	0.9122	0.8315	0.8316
1	1.5871	1.5872	1.0441	1.0442	0.9469	0.947
2	1.7136	1.7137	1.1547	1.1548	1.0474	1.0475

3	1.8155	1.8156	1.2437	1.2438	1.1300	1.1301
4	1.9016	1.9017	1.3188	1.3189	1.2006	1.2007
5	1.9768	1.9769	1.3843	1.3844	1.2625	1.2626
6	2.0438	2.0439	1.4425	1.4426	1.3179	1.318
7	2.1046	2.1047	1.4950	1.4951	1.3681	1.3682
8	2.1602	2.1603	1.5430	1.5431	1.4142	1.4143
9	2.2116	2.2117	1.5873	1.5874	1.4567	1.4568
10	2.2596	2.2597	1.6285	1.6286	1.4964	1.4965
11	2.3045	2.3046	1.6671	1.6672	1.5335	1.5336
12	2.3469	2.347	1.7033	1.7034	1.5685	1.5686
13	2.3870	2.3871	1.7376	1.7377	1.6015	1.6016
14	2.4250	2.4251	1.7701	1.7702	1.6329	1.633
15	2.4614	2.4615	1.8010	1.8011	1.6628	1.6629
16	2.4961	2.4962	1.8305	1.8306	1.6914	1.6915
17	2.5293	2.5294	1.8588	1.8589	1.7187	1.7188
18	2.5613	2.5614	1.8859	1.886	1.7450	1.7451
19	2.5921	2.5922	1.9120	1.9121	1.7702	1.7703
20	2.6217	2.6218	1.9371	1.9372	1.7946	1.7947
21	2.6504	2.6505	1.9613	1.9614	1.8180	1.8181
22	2.6782	2.6783	1.9848	1.9849	1.8407	1.8408
23	2.7050	2.7051	2.0074	2.0075	1.8627	1.8628
24	2.7311	2.7312	2.0294	2.0295	1.8840	1.8841
25	2.7564	2.7565	2.0507	2.0508	1.9047	1.9048
26	2.7811	2.7812	2.0715	2.0716	1.9248	1.9249
27	2.8050	2.8051	2.0916	2.0917	1.9443	1.9444
28	2.8284	2.8285	2.1113	2.1114	1.9633	1.9634
29	2.8512	2.8513	2.1304	2.1305	1.9819	1.982
30	2.8734	2.8735	2.1490	2.1491	1.9999	2

CONCLUSION

In the present chapter the stability of Eringen micropolar fluid saturated in an isotropic porous layer obeying Darcy's law has been examined. Analytical and numerical analysis provide the following results

For $\mathbf{J_\mu > 0}$

- the stability of non-oscillatory modes, if $\frac{\varepsilon(1+K)}{k_1AK} \geq 1$
- the bounds for arbitrary non-oscillatory unstable modes if $\frac{\varepsilon(1+K)}{k_1AK} < 1$
- the semi-circular bounds for oscillatory unstable modes
- the increase in the range of oscillatory unstable region with the increase in permeability parameter k_1 and micropolar parameter K
- destabilizing character of micropolar parameter K.

For $\mathbf{J_\mu < 0}$

- stability of oscillatory modes
- instability of non-oscillatory modes
- stabilizing character of micropolar parameter K
- destabilizing character of permeability parameter k_1.

This is important to note that micropolar parameter which has stabilizing character for a continuous medium has dual character for a fluid contained in a porous medium.

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