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**INVESTIGATION OF SUPPLY CHAIN MANAGEMENT THROUGH SELLING PRICE  
DEPENDENT DEMAND RATE**

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**ABSTRACT**

Supply chain management involves the entire process of planning, implementing and controlling supply chain operations. It is not just the process of order goods and receiving them into inventory, but making certain that they are shipped and delivered to customers in a timely fashion. That means that those in the procurement areas of each company are responsible for all aspects of goods movement beginning with the purchase requisition and ending with the delivery of finished goods to the customer. In the case of a manufacturing company, this process will also involve the procurement of the raw goods and work-in-process phase of the manufacturing process. Supply chain has become a vital topic in management science and industry. The logical progression of the inventory model is to investigate the supply chain that consists of suppliers, manufacturers, distributors and retailers. Each one of them holds inventory in some form to support the requirement of the customer at the end of the chain. In supply chain many problems still need a careful consideration regarding solution procedure to support respective systems.

**Key words:** Supply chain management, implementing

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**INTRODUCTION**

In recent years, integrating traditional inventory management with other types of decisions made by the firm (e.g., pricing, quality level, guaranty period, etc) has attracted the attention of many researchers because these decisions must be compatible to each other in order to obtain maximal profit. In fact, setting prices and planning for how much inventory to hold are the two most strategic ones among the many decisions made by a manager. Keeping these facts in mind, practitioners and academics have focused on determining pricing strategy, which influences demands, and production-inventory decisions, which define the cost of satisfying those demands, simultaneously. The seminal work in this line of research is by Whitin (1955). He considered the economic order quantity (EOQ) model with pricing for a buyer that has a price dependent demand with a linear function. His work encouraged many researchers to investigate joint pricing and ordering problems. The focus of these models has been on demand functions (e.g., Rosenberg (1991), Lau and Lau (2003), on quantity discount (e.g., Burwell, Dave, Fitzpatrick and Roy (1997), Lin and Ho (2011), or on perishable inventories (e.g., Roy (2008), Khanra, Sana and Chaudhuri (2010)), among others. Chung and Wee (2008) developed joint pricing and ordering problems in another line of research in which multiple companies in a supply chain cooperate with each other. Actually, he inspired the idea of his work from Goyal (1976), which was the first study in the integrated vendor-buyer inventory models. The integrated inventory models, where the total cost of the supply chain is minimised, were developed to overcome the weakness of the traditional inventory management systems in which the members of the supply chain make their own optimal decisions independently, which may not be optimal for the whole system. Many researchers, such as Banerjee (1986), Hill (1997), Ouyang, Wu and Ho (2004), Rad, Khoshalhan and Tarokh (2011), Rad and Khoshalhan (2011) have then extended the work of Goyal (1976). Sajadieh and Jokar (2009) provided an integrated production-inventory-marketing model in which the optimal ordering, pricing and shipment policy are simultaneously determined to maximize the joint total profit of both the vendor and the buyer. Recently, Kim, Hong and Kim (2011) discussed joint pricing and ordering policies for price-dependent demand in a supply chain consisting of a single retailer and a single manufacturer. Some other researchers such as Ho, Ouyang and Su (2008), Chen and Kang (2010) and Chung

and Liao (2011) also developed integrated inventory models that involve price-sensitive demands. The main focus of these works were on trade credit policies and they considered flexible production rates by assuming that the production rate can be varied in the fixed ratios of the demand rate. We refer the readers to comprehensive reviews of joint operations-marketing models were done by Eliashberg and Steinberg (1993), Chan, Shen, Simchi-Levi and Swann (2004), Yano and Gilbert (2005) and Soon (2011) for more studies.

## REVIEW OF LITERATURE

Lead time can be seen in manufacturing process and supply chain management. It is often observed that the manufacturer needs some time to fulfill an order after receiving it. This time is said to be lead time. Each firm desires a reduction in time that is consumed to deliver an item in the market. In business, vendor and buyer generally prefer lead time minimization. The decrease of lead time is important in the case for which customer's demand is in fluctuable state and varies with respect to time, since more lead time can put the industries at a risk of storages before the arrival of the items. Liao and Shyu (1991) initially worked on variable lead time to develop an inventory model for fixed order quantity and normally distributed demand. Researchers developed the inventory model to reduce lead time considering different components which can be reduced up to a predefined minimum duration that helps the piecewise linear crashing cost functions. In this method by reducing the lead time total cost can be decreased. Kim and Benton (1995) worked on stochastic continuous review inventory model that is a linear relationship between lot size and lead time. Ouyang et al (1996) established a model to lower the safety stock reduce the loss due to stock out, improve customer service level and increase better ability in business by shortening the lead time, the shortages are also considered in this model and are partially backlogged, a fraction of demand during stock out is assumed to be a lost sale. Moon and Choi (1998) generalized a model for the mixture of backorders and lost sale that are order quantity, the reorder time and lead time are decision variables. Hariga and Daya (1999) worked out an inventory model with complete and partial backlogging under lead time. In this discussed model Hariga and Daya (1999) considered lot-size, reorder point and lead time as decision variables in their inventory model. Pan and Yang (2002) established an economic order quantity (EOQ) model with fixed and dynamic lead time for crashing costs. In this model lead time can be reduced at the expense of additional cost. Wu and Lui (2004) presented an EOQ model with lead time under the consideration that the quantity received at the arrival of the stock may be different from the ordered quantity. In this method Wu and Lui (2004) proved that the lead time can be reduced by considering crashing cost. Hsu et al (2007) proposed a model for deteriorating items with uncertain lead time having certain expiry date. In this model demand is considered as seasonal, price sensitive with shortages and supplier's lead time is taken as random variable depending on the managing cost. Tyagi (2016) investigated an EOQ model for cost minimization to establish the retailer's optimal inventory cycle time and optimal order quantity.

## ASSUMPTIONS AND NOTATIONS

The mathematical model in this study is developed on the basis of the following assumptions:

- The production rate is finite and is greater than the sum of all the buyer demand.
- There is no replacement or repair of deteriorated units.
- The cost of a backorder includes a fixed cost and a cost is proportional to the length of time for which backorder exist.
- The cost of a lost sale, excluding the lost of profit, is constant (goodwill cost).

The following notation is assumed:

$\theta$	The constant deterioration rate ( $0 < \theta < 1$ )
D	Demand rate for vendor, $D = \alpha P_v^\beta$ where c, d are positive constants
R	Demand rate for buyer, $R = \alpha P_b^\beta$ where c, d are positive constants
KD	The production rate per year, where $K > 1$
T	Time length of each cycle, where $T = T_1 + T_2$

$T_1$	The length of production time in each production cycle
$T_2$	The length of non production time in each production cycle.
$I_{v1}(t_1)$	Inventory level for vendor when $t_1$ is between 0 and $T_1$
$I_{v2}(t_2)$	Inventory level for vendor when $t_2$ is between 0 and $T_2$
$I_b(t)$	Inventory level for buyer when $t$ is between 0 and $t_1$
$I_{b1}(t)$	Inventory level for buyer when $t$ is between $t_1$ and $T/n$
$I_{mv}$	The maximum inventory level for vendor
$I_{mi}$	The maximum inventory level for buyer
$S_v$	The setup cost for each production cycle for vendor
$S_b$	The setup cost per order for buyer
$(F_v + \Phi t)$	Holding cost per unit time for vendor
$(F_b + \Phi t)$	Holding cost per unit time for buyer
$C_v$	Deterioration cost per unit time for vendor
$C_b$	Deterioration cost per unit time for buyer
$K_b$	Shortage cost per unit time for buyer
$O_b$	Opportunity cost per unit time for buyer
$P_v$	Vendor's retail price
$p_v$	The unit production cost for vendor
$p_b$	The unit price for buyer
$w_0$	Fixed shortage cost per/ independent time ( $\geq 0$ )
$w$	Fixed shortage cost per unit backordered
$\pi_0$	Goodwill cost of a lost sale, that is, the cost derived of a Lost sale excluding the lost of profit ( $\geq 0$ )
$\psi$	Length of the inventory cycle over which the net stock is less than or equal to zero, that is, length of shortage cycle ( $\geq 0$ )
VC	The cost of vendor per unit time
BC	The cost of buyer per unit time
TC	The integrated cost of vendor and all buyer per unit time

## MATHEMATICAL ANALYSIS

### The Model for Single vendor and Single buyer

The vendor inventory model

The cycle time interval is  $T$ , it can be divided into two periods: the production period during  $T_1$  and the non-production period during  $T_2$ .

The inventory system is represented by the following differential equations:

$$I'_{v1}(t_1) + \theta I_{v1}(t_1) = \alpha P_v^\beta (K - 1) \quad , 0 \leq t \leq T_1 \quad \dots(1)$$

$$I'_{v2}(t_2) + \theta I_{v2}(t_2) = -\alpha P_v^\beta \quad , 0 \leq t \leq T_2 \quad \dots(2)$$

Various boundary conditions  $I_{v1}(0) = 0, I_{v2}(T_2) = 0$ , the solutions of the above differential equations are

$$I_{v1}(t_1) = \frac{(K - 1)\alpha P_v^\beta}{\theta} (1 - e^{-\theta t_1}) \quad , 0 \leq t_1 \leq T_1 \quad \dots(3)$$

$$I_{v2}(t_2) = \frac{\alpha P_v^\beta}{\theta} \left( \frac{e^{\theta T_2} - e^{\theta t_2}}{e^{\theta t_2}} \right) \quad , 0 \leq t_2 \leq T_2 \quad \dots(4)$$

From (4)

$$I_{mv} = \frac{\alpha P_v^\beta}{\theta} (e^{\theta T_2} - 1) \quad \dots(5)$$

When  $I_{mv} = I_{v2}(0)$

By the boundary condition,  $I_{v1}(T_1) = I_{v2}(0)$ , one can drive the following equation:

$$T_1 = \frac{1}{K-1} T_2 \quad \dots(6)$$

Knowing

$$T = T_1 + T_2 \quad \dots(7)$$

one can derive

$$T = \frac{K}{K-1} T_2 \quad \dots(8)$$

### The buyer inventory model

In this article, the inventory system goes like this:  $I_{mi}$  units of item arrive at the inventory system at the beginning of each cycle. During the time interval  $[0, t_1]$ , the inventory level is dropping to zero due to demand and deterioration. Then shortage interval keeps to the end of the current order cycle.

During the time interval  $[0, t_1]$ , the interval level decreases owing to price sensitive demand rate as well as deterioration. Thus, the differential equation representing the inventory status is given by

$$I_b'(t) + \theta I_b(t) = -\alpha P_b^\beta \quad , 0 \leq t \leq t_1 \quad \dots(9)$$

with the boundary conditions  $I_b(t_1) = 0$ . The solution of Eq. (9) is

$$I_b(t) = \frac{\alpha P_b^\beta}{\theta} \left( e^{\theta(t_1-t)} - 1 \right) \quad , 0 \leq t \leq t_1 \quad \dots(10)$$

From (10)

$$I_{mi} = \frac{\alpha P_b^\beta}{\theta} \left( e^{\theta t_1} - 1 \right) \quad \dots(11)$$

During the shortage interval  $\left[ t_1, \frac{T}{n} \right]$ , the demand at time  $t$  is partially backlogged at fraction  $B\left(\frac{T}{n} - t\right)$ . Thus, the inventory level at time  $t$ , is governed by the following differential equation:

$$I_{b1}'(t) = \frac{-\alpha P_b^\beta}{1 + \delta\left(\frac{T}{n} - t\right)} \quad t_1 \leq t \leq \frac{T}{n} \quad \dots(12)$$

with the boundary conditions  $I_{b1}(t_1) = 0$ . The solution of Eq. (12) is

$$I_{b1}(t) = \frac{-\alpha P_b^\beta}{\delta} \left\{ \ln \left[ 1 + \delta\left(\frac{T}{n} - t_1\right) \right] - \ln \left[ 1 + \delta\left(\frac{T}{n} - t\right) \right] \right\} \quad , t_1 \leq t \leq \frac{T}{n} \quad \dots(13)$$

Putting  $t = \frac{T}{n}$  in Eq. (16), we obtain the maximum amount of demand backlogged per cycle as follows:

$$S = -I_{b1}\left(\frac{T}{n}\right) = \frac{\alpha P_b^\beta}{\delta} \ln \left[ 1 + \delta\left(\frac{T}{n} - t_1\right) \right] \quad \dots(14)$$

From Eq. (11) and (14), we can obtain the order quantity,  $Q$ , as

$$Q = I_{mi} + S \\ = \frac{\alpha P_b^\beta}{\theta} \left( e^{\theta t_1} - 1 \right) + \frac{a}{\delta} \ln \left[ 1 + \delta\left(\frac{T}{n} - t_1\right) \right] \quad \dots(15)$$

Next, the relevant inventory cost per cycle for vendor consists of the following three elements:

The setup cost per cycle is

$$S_v \quad \dots(16)$$

The inventory holding cost per cycle is

$$HC_v = p_v \left[ \int_0^{T_1} (F_v + \phi t) I_{v1}(t) dt + \int_0^{T_2} (F_v + \phi t) I_{v2}(t) dt - \int_0^{t_1} (F_v + \phi t) I_b(t) dt \right] \quad \dots(17)$$

The deterioration cost per cycle is

$$DC_v = \theta C_v \left[ \int_0^{T_1} I_{v1}(t) dt + \int_0^{T_2} I_{v2}(t) dt - \int_0^{t_1} I_b(t) dt \right] \quad \dots(18)$$

The purchase cost per cycle is

$$PC_v = p_v K \alpha P_v^\beta T_1 \quad \dots(19)$$

The vendor's total cost is the sum of (16), (17), (18), (19) and (26) as

$$VC = \text{holding cost} + \text{deterioration cost} + \text{ordering cost} + \text{purchase cost} - \text{Buyer's purchase cost} \quad \dots(20)$$

Next, the relevant inventory cost per cycle for buyer consists of the following five elements:

The setup cost per cycle is

$$nS_b \quad \dots(21)$$

The inventory holding cost per cycle is

$$HC_b = p_b \int_0^{t_1} (F_b + \phi t) I_b(t) dt \quad \dots(22)$$

The deterioration cost per cycle is

$$DC_b = \theta C_b \int_0^{t_1} I_b(t) dt \quad \dots(23)$$

The shortage cost per cycle is

$$SC_b = K_b \int_{t_1}^{T/n} [-I_{b1}(t)] dt \quad \dots(24)$$

The opportunity cost per cycle is

$$OC_b = O_b \int_{t_1}^{T/n} \alpha P_b^\beta [1 - B] dt \quad \dots(25)$$

The purchase cost per cycle is

$$PC_b = p_b Q \quad \dots(26)$$

The buyer's total cost is the sum of (21), (22), (23), (24), (25) and (26) as

$$BC = \text{holding cost} + \text{deterioration cost} + \text{shortage cost} + \text{opportunity cost} + \text{ordering cost} + \text{purchase cost} - \text{customer's purchase cost} \quad \dots(27)$$

The integrated joint total cost function TC for the vendor and the buyer is the sum of VC and BC. From (20) and (27),

The integrated total cost can be written as

$$TC = VC + BC \quad \dots(28)$$

As can be plainly observed this is a function of a continuous variable  $T_2, t$

## CONCLUSION

Integrated inventory model for decaying items with price sensitive demand over an infinite-horizon has been developed. There is no doubt that price affect demand. In the case of price elasticity of demand it is used to see how sensitive the demand for a good is to a price change. The higher the price elasticity, the more sensitive consumers are to price changes. Very high price elasticity suggests that when the price of a good goes up, consumers will buy a great deal less of it and when the price of that good goes down, consumers will buy a great deal more. The fraction of backlogged demand is described by a function which depends on the waiting time before receiving the item and on the length of the inventory cycle over which the net stock is not positive. The model has been solved numerically. It is observed that as the difference between vendor's retail price and buyer's retail price increases, total cost increases. As should have been, it proves that less difference in their retail prices to be financially better. The reason being very obvious, that in today's time with so much volatility around, a model with same retail price for both vendor and buyer just does not sound feasible.

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