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# FIXED POINT OF WEAK COMPATIBLE MAPPINGS IN COMPLETE FUZZY METRIC SPACE

**Akhilesh Jain**
Department of Mathematics
Corporate Institute of Science and Technology, Bhopal, M.P.

**Dr. R.S. Chandel**
Department of mathematics
Govt. Geetanjali Girls College
Bhopal, M.P.(India)

**Dr. N. K. Gautam**
Department of Mathematics
Govt. Model Science College
Gwalior, M.P. (India)

**Abstract:**
In this paper, we introduce the concepts of WEAK COMPATIBLE MAPPINGS and prove the common fixed point theorem for weakly compatible mappings on complete fuzzy metric spaces.

**Keywords:**
Intuitionistic fuzzy metric space, Triangular norm, Triangular co-norm, Commutative mappings, Compatible mappings, fixed point.

**AMS Mathematics Subject Classification (2000): 54H25,54A40,47H10**

**Introduction:**

The theory of fuzzy sets has evolved in many directions after investigation of notion of fuzzy sets by Zadeh [36] and is finding many applications in wide variety of fields in which the phenomenon under study are too complex or too ill defined to be analyzed by the conventional techniques. Fuzzy sets were taken up with enthusiasm by engineers, computer scientists and operations researchers, particularly in Japan where fuzzy controllers are now an integral feature of many manufactured devices. In applications of fuzzy set theory the field of engineering has undoubtedly been a leader. All engineering disciplines such as civil engineering, electrical engineering, mechanical engineering, robotics, industrial engineering, computer engineering, nuclear engineering etc. have already been affected to various degrees by the new methodological possibilities opened by fuzzy sets. There is large number of authors who studied applications of fuzzy set theory in different engineering branches. We are mentioning some of them Fetz [5], [6] Fetz et al. [7], Halder and Reddy [11], Lessmann, Muhologger and Oberguggenberger [22] and many others applied fuzzy set theory in civil engineering. A method utilizing the mathematics of fuzzy sets has been shown to be effective in solving engineering problems such as aircraft gas turbine (Law and Antonsson [21]), car body structure NVH design (Mathai and Cronin [24]), multiobjective system optimization (Rao and Dhingra [25]), preliminary passenger vehicle structure (Scott, Law and Antonsson [27]), computational tools for preliminary engineering design (Wood and Antonsson [35]), knowledge base system design (Zimmermann and Sebastian [37]), intelligent system design support (Zimmermann and Sebastian ), machine flexibility (Tsourveloudis et al. [34]) and many others.

A fixed point is one of the basic tools to handle various physical formulations. Fixed point theorems in fuzzy mathematics are emerging with vigorous hope and vital trust. There have been several attempts to formulate fixed point theorems in fuzzy mathematics. From amongst several formulations of fuzzy metric spaces (Deng [47], Erceg [51], Kaleva and Seikkala [19], Kramosil and Michlek [20] George and Veeramani [9]) Grabiec

[8] followed Kramosil and Michalek [20] and obtained fuzzy version of Banach contraction principle. Jungck [14] established common fixed point theorem for commuting maps generalizing the Banach's fixed point theorem. Sessa [26] defined a generalization of commutativity. Further Jungck [15] introduced more generalized commutativity so called compatibility. Jungck and Rhoades [16] introduced the notion of weak compatible maps and proved that compatible maps are weakly compatible but converse is not true.

The notion of compatible maps in fuzzy metric spaces has been introduced by Mishra et al. [23], compatible maps of type (α) by Cho [2] and compatible maps of type ( β) by Cho, Pathak, Kang and Jung [40]. Chang, Cho, Lee, Jung and Kang [44], Cho [1], Fang [60], George and Veeramani [9], Jung, Cho, Chang and Kang [18], Jung, Cho and Kim [17], Mishra, Sharma and Singh [23], Sharma [29], [30], Sharma and Deshpande [32]-[33], Subrahmanyam [28] and many others studied fixed point theorems in fuzzy metric spaces. Various fixed point theorems, for compatible mappings satisfying contractive type conditions and assuming continuity of at least one of the mappings in the compatible pairs in metric spaces and fuzzy metric spaces, have been obtained by many authors.

**Preliminaries:**

Now we begin with some definitions

**Definition** 1 : A binary operation * : $[0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if ([0,1], *) is an Abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, d are in [0,1].

Examples of t-norm are a * b = ab and a * b = min {a,b}.

**Definition 2**: The 3-tuple (X, M, *) is called a fuzzy metric space (shortly FM-space) if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty)$ satisfying the following conditions for all x, y, z in X and t, s > 0,

(FM-1) M(x, y, 0) = 0,
(FM-2) M(x, y, t) = 1 for all t > 0 if and only if x = y,
(FM-3) M(x, y, t) = M(y, x, t),
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$,
(FM-5) $M(x, y,.): [0,1] \rightarrow [0,1]$ is left continuous.

In what follows, (X, M, *) will denote a fuzzy metric space. Note that M(x, y, t) can be thought as the degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t) = 1 for all t > 0 and M(x, y, t) = 0 with ∞ and we can find some topological properties and examples of fuzzy metric spaces in (George and Veeramani [9]).

**Example** 1: Let (X, d) be a metric space. Define a * b = ab or a * b = min {a, b} and for all x, y in X and t > 0,

Then (X, M, * ) is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

In the following example, we know that every metric induces a fuzzy metric.

**Lemma** 1: For all x, $y \in X$, M(x, y,.) is non-decreasing.

**Definition 3** : Let (X, M, *) be a fuzzy metric space .

A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim x_n = x$), if

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1, \qquad \text{for all } t > 0.$$

**Remark 1**: Since * is continuous, it follows from (FM-4) that the limit of the sequence in FM-space is uniquely determined.

Let (X,M,*) be a fuzzy metric space with the following condition:

(FM-6) $\lim_{t \to \infty} M(x,y,t) = 1$ for all $x,y \in X$.

**Lemma 2** : If for all $x,y \in X$, $t > 0$ and for a number $k \in (0,1)$,

$M(x,y,kt) \geq M(x,y,t)$ then $x = y$.

**Lemma 3** : Let $\{y_n\}$ be a sequence in a fuzzy metric space (X, M,*) with the condition (FM-6). If there exists a number $k \in (0,1)$ such that

$$M(y_{n+1},y_{n+2},kt) \geq M(y_n,y_{n+1},t) \quad (1.b)$$

for all $t > 0$ and $n = 1,2,...$ then $\{y_n\}$ is a Cauchy sequence in X.

**Definition 4**: Let S and T be mappings from a fuzzy metric space (X, M,*) into itself. The mappings S and T are said to be compatible if

$$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1,$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X.$$

**Definition 5** : Let S and T be mappings from a fuzzy metric space (X,M,*) into itself. The mappings S and T are said to be compatible of type ($\alpha$) if,

$\lim_{n\to\infty} M(STx_n, TTx_n, t) = 1$, $\lim_{n\to\infty} M(TSx_n, SSx_n, t) = 1$,

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

**Definition** 6 : A pair of mappings S and T is called weakly compatible pair in fuzzy metric space if they commute at coincidence points; i.e. , if $Tu = Su$ for some $u \in X$, then $TSu = STu$.

**Example 2:** Let $X = [0, 2]$ with the metric d defined by $d(x, y) = |x - y|$. For each $t \in (0, \infty)$ define

$M(x, y, t) = t / t+ d(x,y)$ $\quad x, y \in X$

$M(x, y, 0) = 0$ $\quad x, y \in X$

Clearly M(x, y, *) is a fuzzy metric space on X where * is defined by $a*b = ab$ or $a * b = \min \{a, b\}$.

Define $A, B : X \to X$ by

$Ax = x$ if $x \in [0, {}^1\!/_3)$, $A(x) = {}^1\!/_3$ if $x \geq {}^1\!/_3$ and $Bx = x/(x+1)$ for all $x \in [0, 2]$

Consider the sequence $\{x_n = (½) + ({}^1\!/_n) : n \geq 1\}$ in X .

Then $\lim_{n \to \infty} Ax_n = {}^1\!/_3$, $\lim_{n \to \infty} Bx_n = {}^1\!/_3$.

But $\lim_{n \to \infty} M(ABx_n, BAx_n, t) = t / t+[(1/3)-(1/4)] \neq 1$.

Thus A and B are noncompatible. But A and B are commuting at their coincidence point $x = 0$, that is, weakly compatible at $x = 0$.

Also $\lim_{n \to \infty} M(ABx_n, BBx_n, t) = t / t+[(1/3)-(1/4)] \neq 1$ and

$\lim_{n \to \infty} M(BAx_n, AAx_n, t) = t / t+[(1/4)-(1/3)] \neq 1$

Thus A and B are not compatible of type($\alpha$).

Further,

$\lim_{n \to \infty} M(AAx_n, BBx_n, t) = t / t+[(1/3)-(1/4)] \neq 1$

Thus A and B are **not compatible** of type ($\beta$).

In view of this example, we observe that weakly compatible maps need not be compatible, weakly compatible maps need not be compatible of type ($\alpha$), weakly compatible maps need not be compatible of type ($\beta$).

We prove the following.

**Theorem** : Let (X, M, *) be a complete fuzzy metric space with continuous t-norm * defined by $(t*t) \geq t$ for all $t \in [0,1]$ and A, B, S, and T be mappings from X into itself satisfying the following conditions

(1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,

(1.2) $M(Ax,By,kt) \leq \varphi[((M^2(Sx,Ty,t),. (M(Sx,By,(2-\alpha)t)*M(Ty,Ax,t))^2,$
$(M(Sx,Ax,t)*M(Ty,Ax,t))^2]^{1/2}$ ,

for all $x,y \in X$ and $\alpha \in [0,2]$ and $t > o$ .

(1.3) the pairs {A,S} and {B,T} are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Throughout this paper $R^+$ denotes the set of non-negative real numbers and the function $\varphi(R^+)^3 \to R^+$ satisfies the following conditions:

(i) $\varphi$ is upper semi-continuous, non-decreasing in each coordinate variable ;

(ii) for each $t > 0$, $\gamma(t) = \varphi(t,t,t) < t$, where $\gamma : R^+ \to R^+$ .

**Main Results:**

We extend Theorem A for five mappings with a different proof. We prove the following.

**Theorem 1** : Let (X, M, *) be a complete fuzzy metric space with continuous t-norm * defined by $(t*t) \geq t$ for all $t \in [0,1]$ and A, B, S, T and P be mappings from X into itself satisfying the following conditions

(1.1) $P(X) \subset AB(X)$ and $P(X) \subset ST(X)$,

(1.2) There exists a number $k \in (0,1)$ such that

$M(Px,Py,kt) \geq \varphi[M^2(ABx,Px,t),(M(STy,Py,t)*M(STy,Px,\alpha t))^2,$
$(M(ABx,Py,(2-\alpha)t)*M(ABx,STy,t))^2]^{1/2}$ ,

for all $x,y \in X$ and $\alpha \in (0,2)$ and $t > o$ .

(1.3) AB = BA , ST = TS , PB = BP , PT = TP ,

(1.4) the pairs {P,AB} and {P,ST} are weakly compatible.

Then A, B, S ,T and P have a unique common fixed point in X.

**Proof** : By (1.1) since $P(X) \subset AB(X)$ , for any point $x_0 \in X$ , there exists a point $x_1 \in X$ such that $Px_0 = ABx_1$ . Since $P(X) \subset ST(X)$, for this point $x_1$ ,we can choose a point $x_2 \in X$ such that $Px_1 = STx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

(1.5) $y_{2n} = Px_{2n} = ABx_{2n+1}$ and
$y_{2n+1} = Px_{2n+1} = STx_{2n+2}$ for $n = 0,1,2,...$

Now we show that $\{y_n\}$ is a Cauchy sequence. For $t > 0$ and $\alpha = 1-q$ with $q \in (0,1)$, firstly we prove that

$M(y_{2n+1},y_{2n+2},kt) \geq M(y_{2n},y_{2n+1},t)$ if
$M(y_{2n+1},y_{2n+2},kt) < M(y_{2n},y_{2n+1},t)$

then by (1.2), we have

$M(y_{2n+1},y_{2n+2},kt) = M(Px_{2n+1},Px_{2n+2},kt)$
$\geq \varphi[M^2(ABx_{2n+1},Px_{2n+1},t),$
$(M(STx_{2n+2},Px_{2n+2},t)*M(STx_{2n+2},Px_{2n+1},\alpha t))^2,$
$(M(ABx_{2n+1},Px_{2n+2},(2-\alpha)t)*M(ABx_{2n+1},STx_{2n+2},t))^2]^{1/2}$

Putting $\alpha = 1-q$ , we have

$\geq \varphi[M^2(y_{2n},y_{2n+1},t),(M(y_{2n+1},y_{2n+2},t)*M(y_{2n+1},y_{2n+1},(1-q)t))^2,(M(y_{2n},y_{2n+2},(1+q))t)*M(y_{2n},y_{2n},t))^2]^{1/2}$

$\geq \varphi[M^2(y_{2n},y_{2n+1},t),(M(y_{2n+1}, y_{2n+2}, t) *1 )^2,$
$(M(y_{2n},y_{2n+1}, t)* M(y_{2n+1},y_{2n+2}, qt)* M(y_{2n},y_{2n+1}, t)))^2]^{1/2}$

$\geq \varphi[M^2(y_{2n},y_{2n+1},t),(M(y_{2n+1}, y_{2n+2}, t))^2,$
$(M(y_{2n},y_{2n+1}, t)* M(y_{2n+1},y_{2n+2}, qt))^2]^{1/2}$

Since t-norm * is continuous and M(x,y, .) is left continuous, letting $q \to 1$, we have

$M(y_{2n+1},y_{2n+2}, kt) \geq \varphi[M^2(y_{2n},y_{2n+1},t),(M(y_{2n+1}, y_{2n+2}, t))^2,$
$(M (y_{2n},y_{2n+1}, t)* M(y_{2n+1},y_{2n+2}, t))^2]^{1/2}$ ,

for n = 1,2, . . . and so , for positive integers n, p,

$M(y_{2n+1},y_{2n+2}, kt) \geq \varphi[M^2(y_{2n},y_{2n+1},t),(M(y_{2n+1}, y_{2n+2}, t/k^p))^2,$
$(M(y_{2n},y_{2n+1}, t)* M(y_{2n+1},y_{2n+2}, t/k^p))^2]^{1/2}$ ,

Thus since $M (y_{2n+1},y_{2n+2}, t/k^p) \to 1$ as $p \to \infty$, we have

$M(y_{2n+1},y_{2n+2}, kt) \geq \varphi[((M^2(y_{2n},y_{2n+1},t),1,(M(y_{2n},y_{2n+1}, t)))^2]^{1/2}$ ,

Since φ is a non-decreasing in each coordinate variable, we claim that for every $n \in N$,

$M(y_{2n+1},y_{2n+2}, kt) \geq M(y_{2n},y_{2n+1}, t)$

For if $M(y_{2n+1},y_{2n+2}, t) < M(y_{2n},y_{2n+1}, t)$ , then by (1.2) , we have

$M(y_{2n+1},y_{2n+2}, kt) \geq \varphi[M^2(y_{2n},y_{2n+1},t),1,(M(y_{2n},y_{2n+1}, t)))^2]^{1/2}$ ,

$M(y_{2n+1},y_{2n+2}, kt) \geq \gamma [M^2(y_{2n}, y_{2n+1}, t))]^{1/2} < M(y_{2n}, y_{2n+1}, t).$

This gives

$M(y_{2n},y_{2n+1}, kt) < M(y_{2n}, y_{2n+1}, t).$

This is a contradiction. Hence $M(y_{2n+1},y_{2n+2}, kt) \geq M(y_{2n}, y_{2n-1}, t)$, for every $n \in N$ and $t > 0$ .

Similarly, we have $M(y_{2n+2},y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t).$

Thus $\{ M(y_{2n+1},y_{2n+2}, kt)\}$ is an increasing sequence in [0,1].

Hence by Lemma 3, $\{y_n\}$ is a Cauchy sequence and by the completeness of the space X, $\{y_n\}$ converges to z. Since $\{Px_{2n}\}$ , $\{ABx_{2n+1}\}$,$\{STx_{2n+2}\}$ are sub sequences of $\{y_n\}$also converges to z.

Similarly since $P(X) \subset AB(X)$ , there exists a point $u \in X$ such that $u = (AB)^{-1}z$. Then ABu = z. We shall use the fact that the subsequence of $\{y_n\}$ also converges to z. By (1.2), with $\alpha = 1$, we have

$M(Pu,Px_{2n+2}, kt) \geq \varphi [M^2(ABu,Pu,t),$
$(M(STx_{2n+2}, Px_{2n+2}, t) * M(STx_{2n+2}, Pu, t))^2,$
$(M(ABu,Px_{2n+2},t)*M(ABu,STx_{2n+2},t))^2]^{1/2}$ ,

$M(Pu,Px_{2n+2}, kt) \geq \varphi [M^2(z,Pu,t),$
$(M(STx_{2n+2}, Px_{2n+2}, t) * M(STx_{2n+2}, Pu, t))^2,$
$(M(z,Px_{2n+2},t)*M(z,STx_{2n+2},t))^2]^{1/2}$

which implies that, as $n \to \infty$ ,

$M(Pu, z, kt) \geq \varphi [M^2(z ,Pu,t),$
$(M(z,z,t)*M(z,Pu,t))^2, (M(z, z,t)*M(z , z ,t))^2]^{1/2}$

Or $M (Pu, z, kt) \geq \varphi [M^2(z ,Pu,t), M^2(z , Pu, t), 1]^{1/2}$

Or $M (Pu, z, kt) \geq \gamma [M^2 (z, Pu, t))]^{1/2} < M(z, Pu, t)$

I.e.

$M(Pu, z, kt) < M(z, Pu, t)$

which is a contradiction. Therefore Pu = z. Thus Pu = ABu = z.

Similarly since $P(X) \subset ST(X)$, there exists a point $v \in X$ such that $STv = z$. Then by (1.2) with $\alpha = 1$, we have

$$M(Pu,Pv,kt) \geq \varphi [M^2(ABu,Pu,t),(M(STv,Pv,t) * M(STv,Pu,t))^2, (M(ABu,Pv,t)*M(ABu,STv,t))^2]^{1/2}$$

or $M(z,Pv,kt) \geq \varphi [M^2(z,z,t),(M(z,Pv,t) * M(z,z,t))^2, (M(z,Pv,t)*M(z,z,t))^2]^{1/2}$

or $M(Pu,Pv,kt) \geq \varphi [M^2(z,z,t),(M(z,Pv,t) * M(z,z,t))^2, (M(z,Pv,t)*M(z,z,t))^2]^{1/2}$

$$M(z,Pv,kt) \geq \gamma [M^2(z,Pv,t)]^{1/2} < M(z,Pv,t),$$

$$M(z,Pv,kt) < M(z,Pv,t),$$

which is a contradiction. Therefore $z = Pv$. Thus $STv = Pv = z$.

Hence $ABu = STv = Pu = Pv = z$.

Since the mappings AB and P are weakly compatible therefore, commute at their coincidence point i.e. $(AB)Pu = P(AB)u$ i.e. $ABz = Pz$.

Now for $\alpha = 1$ in (1.2), we have

$$M(Pz,Px_{2n+2},kt) \geq \varphi [M^2(ABz,Pz,t), (M(STx_{2n+2},Px_{2n+2},t) * M(STx_{2n+2},Pz,t))^2, (M(ABz,Px_{2n+2},t)*M(ABz,STx_{2n+2},t))^2]^{1/2}$$

Or $M(Pz,Px_{2n+2},kt) \geq \varphi [M^2(Pz,Pz,t), (M(STx_{2n+2},Px_{2n+2},t) * M(STx_{2n+2},Pz,t))^2, (M(Pz,Px_{2n+2},t)*M(Pz,STx_{2n+2},t))^2]^{1/2}$

which implies that, as $n \to \infty$,

$$M(Pz,z,kt) \geq \varphi [1,M^2(z,Pz,t), (M(Pz,z,t)*M(Pz,z,t))^2]^{1/2}$$

$$M(Pz,z,kt) \geq \varphi [M^2(z,Pu,t), M^2(z,Pu,t), 1]^{1/2}$$

Or $M(Pz,z,kt) \geq \gamma[M^2(z,Pz,t))]^{1/2} < M(z,Pz,t)$

i.e. $M(Pz,z,kt) < M(z,Pz,t)$

which is a contradiction. Therefore $Pz = z$. Thus $Pz = ABz = z$.

Similarly since the mappings ST and P are weakly compatible therefore, commute at their coincidence point i.e. $(ST)Pu = P(ST)u$ i.e. $STz = Pz$.

Now for $\alpha = 1$ in (1.2), we have

$$M(Pu,Pz,kt) \geq \varphi [M^2(ABu,Pu,t), (M(STz,Pz,t) * M(STz,Pu,t))^2, (M(ABu,Pz,t)*M(ABu,STz,t))^2]^{1/2},$$

$$M(z,Pz,kt) \geq \varphi [M^2(z,z,t),(M(Pz,Pz,t) * M(Pz,z,t))^2, (M(z,Pz,t)*M(z,Pz,t))^2]^{1/2}$$

$$M(z,Pz,kt) \geq \varphi [1, M^2(Pz,z,t),M^2(z,Pz,t))^2]^{1/2}$$

$$M(Pz,z,kt) \geq \gamma[M^2(z,Pz,t))]^{1/2} < M(z,Pz,t)$$

i.e. $M(Pz,z,kt) < M(z,Pz,t)$

which is a contradiction. Therefore $Pz = z$. Thus $Pz = STz = z$.

This gives $Pz = STz = ABz = z$.

By taking $x = Bz$ and $y = x_{2n+2}$ in (1.2) using (1.3) with $\alpha = 1$, we have

$$M(P(Bz),Px_{2n+2},kt) \geq \varphi [M^2(AB(Bz),P(Bz),t), (M(STx_{2n+2},Px_{2n+2},t) * M(STx_{2n+2},P(Bz),t))^2, (M(AB(Bz),Px_{2n+2},t)*M(AB(Bz),STx_{2n+2},t))^2]^{1/2}$$

Or $M(Bz,Px_{2n+2},kt) \geq \varphi [M^2(Bz,Bz,t), (M(STx_{2n+2},Px_{2n+2},t) * M(STx_{2n+2},Bz,t))^2, (M(Bz,Px_{2n+2},t)*M(Bz,STx_{2n+2},t))^2]^{1/2}$

which implies that, as $n \to \infty$,

$M(Bz, z, kt) \geq \varphi [ 1, (M(z, z, t) * M(z, Bz, t))^2, (M(Bz, z, t)*M(Bz, z, t))^2]^{1/2}$ ,
$M(Bz, z, kt) \geq \varphi [ 1, (1 * M(z, Bz, t))^2, (M(Bz, z, t)*M(Bz, z, t))^2]^{1/2}$
$M(Bz, z, kt) \geq \gamma[M^2(z, Bz, t))]^{1/2} < M(z, Bz, t)$
I.e. $M(Bz, z, kt) < M(z, Bz, t)$
which is a contradiction. Thus $Bz = z$ . Since $z = ABz$ , we have $z = Az$ . Therefore, $Bz = Az = z = Pz$.
By taking $x = x_{2n+1}$ and $y = Tz$ in (1.2) using (1.3) with $\alpha = 1$, we have
$M(Px_{2n+1}, P(Tz), kt) \geq \varphi[M^2(ABx_{2n+1},Px_{2n+1},t),$
$(M(ST(Tz), P(Tz), t) * M(ST(Tz), Px_{2n+1}, t))^2,$
$(M(ABx_{2n+1},P(Tz),t)*M(ABx_{2n+1},ST(Tz),t))^2]^{1/2}$
$M(Px_{2n+1}, Tz, kt) \geq \varphi[M^2(ABx_{2n+1},Px_{2n+1},t),$
$(M(Tz, Tz, t) * M(Tz, Px_{2n+1}, t))^2,$
$(M(ABx_{2n+1},Tz , t)*M(ABx_{2n+1},Tz , t))^2]^{1/2}$
which implies that, as $n \to \infty$ ,
$M(z , Tz, kt) \geq \varphi[M^2(z , z, ,t), (M(Tz, Tz, t) * M(Tz , z, t))^2,$
$(M(z ,Tz , t)*M(z ,Tz , t))^2]^{1/2}$
Or $M(z , Tz, kt) \geq \varphi[ 1, (1 * M(Tz , z, t))^2, (M(z ,Tz , t)*M(z ,Tz , t))^2]^{1/2}$
Or $M(z , Tz, kt) \geq \gamma[M^2(z, Tz, t))]^{1/2} < M(z, Tz, t)$
i.e.
$M(Tz, z, kt) < M(z, Tz, t)$
which is a contradiction. Thus $Tz = z$ . Since $z = STz$ , we have $z = Sz$ .
$Bz = Az = Pz = Sz = Tz = z$ i.e. z is a common fixed point of A, B, S, T and P.
For the uniqueness let w ( $w \neq z$) be another common fixed point of A, B, S , T and P . Then by (1.2) and using (1.3) with $\alpha = 1$, we have
$M(Pz,Pw, kt) \geq \varphi [M^2(ABz,Pz, t),(M(STw, Pw, t) * M(STw, Pz, t))^2,$
$(M(ABz,Pw, t)*M(ABz,STw,t))^2]^{1/2}$
$M(z,w, kt) \geq \varphi [M^2(z,z, t),(M(w, w, t) * M(w, z, t))^2, (M(z,w, t)*M(z,w,t))^2]^{1/2}$
$M(z,w, kt) \geq \varphi [1,( 1 * M(w, z, t))^2,(M(z,w, t)*M(z,w,t))^2]^{1/2}$ ,
$M(z , w, kt) \geq \gamma[M^2(z, w, t))]^{1/2} < M(z, w, t)$
i.e. $M(w, z, kt) < M(z, w, t)$
which is a contradiction. Therefore, $z = w$ .
This completes the proof.

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