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EXAMINATION OF INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK-DEPENDENT CONSUMPTION RATE

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ABSTRACT

Generally, deterioration is defined as the damage, dryness, spoilage, vaporization, etc., that result in decrease of usefulness of the original one. In real life, mostly goods have a span of maintaining quality or the original condition (e.g. food, vegetables, fish, meat, fruit and so on), define, during that time, there was no deterioration occurring. We have taken a more realistic demand rate that depends on two factors, one is time, and the other is the stock level. The stock level in itself obviously gets depleted due to the customer's demand. As a result, what we witness here is a circle in which the customer's demand is being influenced by the level of stocks available, while the stock levels are getting depleted due to the customer's demands.

Key words: Customer demand, Stock

INTRODUCTION

In the literature of inventory after the development of classical economic order quantity (EOQ) model researchers extensively studied several aspects of inventory modeling by assuming constant demand rate. But in a real market demand of a product is always dynamic state due to the variability of time, price or even of the instantaneous level of inventory displayed in retail shop. This impressed researchers and marketing practitioners to think about the variability of demand rate.

REVIEW OF LITERATURE

Inventory models create a lot of interest due to their ready applicability at various places like market yards, warehouses, production process, transportation systems cargo handling, etc., several inventory models have been developed and analyzed to study various inventory systems. Much work has been reported in literature regarding Economic Production Quantity (EPQ) models during the last two decades. The EPQ models are also a particular case of inventory models. The major constituent components of the EPQ models are 1) Demand 2) production (Production) (Replenishment) and 3) Life time of the commodity. Several EPQ models have been developed and analyzed with various assumptions on demand pattern and life time of the commodity. In general, it is customary to consider that the replenishment is random in production inventory models. Several researchers have developed various inventory models with stock dependent demand. Silver and Peterson (1985) mentioned that the demand for many consumer items is directly proportional to the stock on hand. Gupta and Vrat (1986) have pointed the inventory models with stock dependent demand. Later, Baker and Urban (1988), Mandal and Phaujdhar (1989), Datta and Pal (1990), Venkat Subbaiah, et al. (2004), Teng and Chang (2005), Arya, et al. (2009), Mahata and Goswami (2009a), Panda, et al. (2009c), Roy, et al. (2009), Uma Maheswara Rao, et al. (2010), Yang, et al. (2010), Yang, et al. (2011), Srinivasa Rao and Essay (2012), Jasvinder Kaur, et al. (2013), Santanu Kumar Ghosh, et al. (2015) and others have developed inventory models for deteriorating items with stock dependent demand. In all these models they assumed that the replenishment is instantaneous or having fixed finite rate, except Sridevi, et al. (2010) that developed and analyzed an inventory model with the assumption that the rate of production is random and follows a Weibull distribution. However, in many practical situations arising at production processes, the production (replenishment) rate is dependent on the stock on hand. But in some other situations such as textile markets, seafood's industries, etc., the demand is a function of stock on hand. Levin et al. (1972) has have observed that at times the presence of inventory has a motivational effect on demand. It is also generally known that large pails of goods displayed in the markets encourage customers to buy more.

Thus, in certain items, the demand increases if large amount of stock is on hand. Another important consideration for developing the EPQ models for deteriorating items is the life time of the commodity. For items like food, processing the life time of the commodity is random and follows a generalized Pareto distribution. (Srinivasa Rao, et al. (2005), Srinivasa Rao and Begum (2007), Srinivasa Rao and Eswara Rao (2015)). Very little work has been reported in the literature regarding EPQ models for deteriorating items with random replenishment and generalized Pareto decay having stock dependent demand, even though these models are more useful for deriving the optimal production schedules of many production processes. Hence, in this paper, we develop and analyze an economic production quantity model for deteriorating items with Weibull rate of replenishment and generalized Pareto decay having demand is a function of on hand inventory. The generalized Pareto distribution is capable of characterizing the life time of the commodities which have a minimum period to start deterioration, and the rate of deterioration is inversely proportionate to time.

SUPPLY CHAIN MANAGEMENT

Supply chain management (SCM) is the management of a network of interconnected businesses involved during the product and service packages required by end customers. Supply chain management spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point of origin to point of consumption (supply chain). "design, planning, execution, control, and monitoring of supply chain activities with the objective of creating net value, building a competitive infrastructure, leveraging worldwide logistics, synchronizing supply with demand and measuring performance globally". Supply chain management, is a set of organizations directly linked by one or more of the upstream and downstream flows of products, services, funds, and information from a source to a customer.

A supply chain is a network of interconnected facilities which performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers.

Supply chain management is the integration of key business processes across the supply chain for the purpose of creating value for customers and stakeholders. A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers.

Many manufacturing operations are designed to maximize throughput and lower costs with little consideration for the impact on inventory levels and distribution capabilities. Purchasing contracts are often negotiated with very little information beyond historical buying patterns. The result of these factors is that there is not a single, integrated plan for the organization---there were as many plans as businesses. Clearly, there is a need for a mechanism through which these different functions can be integrated together. Supply chain management is a strategy through which such integration can be achieved.

Supply chain management is typically viewed to lie between fully vertically integrated firms, where the entire material flow is owned by a single firm and those where each channel member operates independently. Therefore coordination between the various players in the chain is key in its effective management.

ASSUMPTIONS AND NOTATIONS

The mathematical models of an inventory problem are based on the following assumptions and notations:

(1) The consumption rate D(t) at any time t is assumed to be $\alpha + \beta I(t) + \gamma t$-*ds*, where α is a positive constant, β is the stock-dependent demand rate parameter, $0 \leq \beta, \gamma \leq 1$, and $I(t)$ is the inventory level at time t.

(2) The replenishment rate is infinite and lead time is zero.

(3) The planning horizon is finite.

(4) Shortages are backlogged at the rate of $e^{-\delta t}$ where $0 < \delta < 1$ and t is the waiting time for next replenishment.

(5) The deterioration rate is $a + b\,t$; $a, b > 0$.

(6) Product transactions are followed by instantaneous cash flow.

Notations:

r discount rate, representing the time value of money
i inflation rate
R *r–i*, representing the net discount rate of inflation is constant
H planning horizon
T replenishment cycle
m the number of replenishments during the planning horizon n = H/T
T_j the total time that is elapsed up to and including the j^{th} replenishment cycle (j=1,2,…..n) where T_0 = 0, T_1 = T, and T_n = H.
t_j the time at which the inventory level in the j^{th} replenishment cycle drops to zero (j=1,2,…..n)
$T_j–t_j$ time period when shortages occur (*j*=1,2,…..n)
Q the 2^{nd}, 3^{rd},…..n^{th} replenishment lot size.
I_m maximum inventory level
A ordering cost per replenishment
C per unit cost of the item
C_h holding cost per unit per unit time
C_s shortage cost per unit per unit time
C_o opportunity cost per unit per unit time due to lost sale.

MODEL FORMULATION

Suppose the planning horizon *H* is divided into *n* equal intervals of length *T* = *H/n*. Hence the reorder times over the planning horizon *H* are $T_j = jT$ (*j*=1, 2,…..n). The period for which there is no shortage in each interval [*jT*, (*j*+1)*T*] is a fraction of the scheduling period *T* and is equal to *kT*, (0 < *k* <1). Shortages occur at time $t_j = (k+j–1)T$, (*j*=1,2,…..n) and are accumulated until time *t*=*jT* (j=1,2,…..n)and shortages are backlogged exponentially.

The first replenishment lot size of I_m is replenished at T_0=0. During the time interval [0, t_1] the inventory level decreases due to stock-dependent demand rate and deterioration and falls to zero at $t = t_1$, now shortages start during the time interval [t_1, *T*] and accumulated until *t* = *T*

The differential equations governing the instantaneous state of inventory level at any time t are given by

$I'(t) + (a+bt)\, I(t) = -[+ \beta I(t)+\gamma t-ds]$ with $I(t_1)=0$ 0 t t_1 …(1)

$I'(t) = -(\alpha + \gamma t-ds)\, e^{-\delta t}$ t_1 to T …(2)

The respective solutions of the above differential equations are

$$I(t) = (\alpha - ds)\exp\left(-(a+\beta)t-\frac{bt^2}{2}\right)\int_t^{t_1}(\alpha - ds + \gamma x)\exp\left((a-ds+\beta)x+\frac{bx^2}{2}\right)dx \quad 0 \ t \ t_1 \qquad \ldots(3)$$

And $$I(t) = -\frac{1}{\delta^2}\left[(\delta\alpha - ds+\gamma)\left(e^{-\delta t_1} - e^{-\delta t}\right)+\delta\gamma\left(t_1e^{-\delta t_1} - te^{-\delta t}\right)\right] \quad t_1 \ t \ T \qquad \ldots(4)$$

The maximum inventory level during first replenishment cycle is

$$I(0) = I_m = \int_0^{t_1}(\alpha - ds+\gamma x)e^{\left((a-ds+\beta)t+\frac{bt^2}{2}\right)}dt \qquad \ldots(5)$$

And the maximum shortage quantity during the first replenishment, which is backlogged

$$I_b = \frac{1}{\delta^2}\left[(\delta\alpha - ds+\gamma)\left(e^{-\delta t_1} - e^{-\delta T}\right)+\delta\gamma\left(t_1e^{-\delta t_1} - Te^{-\delta T}\right)\right] \qquad \ldots(6)$$

The present value of the ordering cost during first replenishment cycle is *A*, as the replenishment is done at the start of each cycle.

The present value of the holding cost of inventory during first replenishment cycle is

$$H.\,C. = C_h\int_0^{t_1} I(t)e^{-Rt}dt \qquad \ldots(7)$$

The present value of the shortage cost during first replenishment cycle is

$$S.\ C. = C_s \int_{t_1}^{T} \frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta t}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - t e^{-\delta t}\right)\right] e^{-Rt} dt \quad \ldots(8)$$

Replenishment items are consumed by demand as well as deterioration during [0, t_1]. The present value of material cost during the first replenishment cycle is

$$C_p = CI_m + \frac{Ce^{-RT}}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta t_1} - e^{-\delta T}\right) + \delta\gamma\left(t_1 e^{-\delta t_1} - T e^{-\delta T}\right)\right] \quad \ldots(9)$$

Opportunity cost due to lost sale

$$O.\ C. = C_o \int_{t_1}^{T} (\alpha - ds + \gamma t)\left(1 - e^{-\delta t}\right) e^{-Rt} dt \quad \ldots(10)$$

Consequently, the present value of total cost of the system during the first replenishment cycle is

$$TRC = A + H.C. + S.C. + C_p + O.C. \quad \ldots(11)$$

The present value of total cost of the system over a finite planning horizon H is

$$TC\,(\text{m}, k) = \sum_{j=1}^{m-1} TRC\ e^{-RjT} - Ae^{-RH} = TRC\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right) - Ae^{-RH} \quad \ldots(12)$$

The present value of total cost TC (n, k) is a function of two variables n and k, where n is a discrete variable and k is a continuous variable. For a given value of n, the necessary condition for $TC\,(m, k)$ to be minimized is $\frac{dTC(m,k)}{dk} = 0$ which gives

$$\frac{C_h H}{m}\left(\alpha - ds + \frac{\gamma kH}{m}\right)\exp\left((a-ds+\beta)\frac{kH}{m} + \frac{b}{2}\left(\frac{kH}{m}\right)^2\right)\int_0^{\frac{kH}{m}} \exp\left(-(a-ds+\beta)t - \frac{bt^2}{2}\right)e^{-Rt}dt$$

$$+\frac{C_s H}{\delta Rm}\left[(\delta\alpha - ds + \gamma)\left\{\exp\left(\frac{(\delta k + R)H}{m}\right) - \exp\left(\frac{(\delta + R)kH}{m}\right)\right\} - \gamma\left(1 - \frac{\delta kH}{m}\right)\right.$$

$$\left.\times\exp\left(-\frac{(\delta k + R)H}{m}\right) + \frac{\gamma}{(\delta + R)}\left\{\left\{\delta - \left(\frac{\delta(\delta + R)kH}{m} - R\right)\right\}\exp\left(-\frac{(\delta + R)kH}{m}\right)\right\}\right]$$

$$+\frac{CH}{m}\left(\alpha - ds + \frac{\gamma kH}{m}\right)\left[\exp\left((a-ds+\beta)\frac{kH}{m} + \frac{b}{2}\left(\frac{kH}{m}\right)^2\right) - \exp\left(\frac{(\delta k + R)H}{m}\right)\right]$$

$$\frac{C_o H}{m}\left(\alpha - ds + \frac{\gamma kH}{m}\right)\left[\exp\left(-\frac{(\delta + R)kH}{m}\right) - \exp\left(-\frac{RkH}{m}\right)\right] = 0 \quad \ldots(13)$$

Provided the condition $\frac{d^2TC(m,k)}{dk^2} > 0$ is satisfied.

We follow the optimal solution procedure proposed by Montgomery (1982), we let (m^*, k^*) denote the optimal solution of $TC(m, k)$ and let $(m, k(m))$denote the optimal solution to $TC(m, k)$ when m is given. If $\tilde{m}$ is the smallest integer such that $TC(\tilde{m}, k(\tilde{m}))$is less than each value of $TC(\tilde{m}, k(\tilde{m}))$ in the interval $\tilde{m}+1 < \tilde{m} < \tilde{m}+10$. Then we take $(\tilde{m}, k(\tilde{m}))$ as the optimal solution to $TC(m, k(m))$. Hence $(\tilde{m}, k(\tilde{m})) = (m^*, k^*)$. Using the optimal procedure described above, we can find the maximum inventory level and optimal order quantity to be

$$I_m = \int_0^{\frac{k^*H}{m^*}} (\alpha - ds + \gamma x)\exp\left((a-ds+\beta)x + \frac{bx^2}{2}\right) dx \quad \ldots(15)$$

And $Q^* = \int_0^{\frac{k^*H}{m^*}} (\alpha - ds + \gamma x)\exp\left((a-ds+\beta)x + \frac{bx^2}{2}\right) dx$

$$+\frac{1}{\delta^2}\left[(\delta\alpha - ds + \gamma)\left(e^{-\delta k^* H/m^*} - e^{-\delta H/m^*}\right) + \delta\gamma\left(\frac{k^*H}{m^*}e^{-\delta k^* H/m^*} - \frac{H}{m^*}e^{-\delta H/m^*}\right)\right] \quad \ldots(16)$$

CONCLUSION

Even till now, most of the researchers have been either completely ignoring the decay factor or are considering a constant rate of deterioration which is not practical. Therefore, we have taken the time dependent decay factor. The problem has been formulated analytically and has been used to arrive at the optimal solution. Also, we could extend the deterministic model into a stochastic model. Finally, we could generalize the model to allow for quantity discounts, trade credits, and others.

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