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A TREATISE ON TOPOLOGICAL b- REGULAR SPACES

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ABSTRACT

The present paper introduces the notion of b- Regular spaces and this concept is important generalisation of regularity using the notion of b- open and b- closed sets in a topological space.

D. Andrejevic introduced a new class of generalised open sets in a topological space, the so called b- open sets, which contains all semi- open sets and pre – open sets. No doubt b- open sets lie in between the class of the union of pre – open sets and semi – open sets and the class of β – open sets. Hence, the framing of this paper bears the main aim to introduce and study b- regularity in the space.

The aim of this paper is to study the class of b- regular spaces and b- T_3space. Many related theorems and examples have been cited. We also obtained characterization theorem and preservation theorem.

KEY WORDS: b – regular space, b- T_3 space, b- homomorphism, b- irresolute, b- closure of set, b- Tychonoffspace.

INTRODUCTION

D .Andrijivic, a polish mathematician, introduced and investigated semi – pre – open sets[1] in 1986. Later on in 1996, he conceptualised b- open sets[2] which are some of the weak forms of open sets. Let A be a subset of a spaeX. Then closure and the interior of A are denoted by cl(A) and int(A) respectively.

DEFINITION (1.1):

A subset A of space (X,T) is called

(a) a pre – open[4,7] set if A int(cl(A) and pre – closed set if cl (int (A))⊆A ;

(b) a semi – open[5] set if A⊆ cl (int(A)) and semi – closed set if int (cl(A)) ⊆ A ;

(c) an α - open set[6] if A ⊆ int (cl(int(A))) and b - closed set if cl(int(cl(A))) ⊆A ;

(d) a β – open set[8] if A ⊆ cl(int(cl(A))) and β – closed set[1] if int (cl(int(A)))⊆A.

Now, a new class ofgeneralized open sets given by D . Andrijevic under the name b – open sets is as below.

DEFINITION (1.2)[2,3] :

A subset A of a space (X,T) is called a b – open set[2] if A ⊆ cl(int(A)) ∪ int (cl(A)) and a b – closed set[3] if cl(int(A)) ∩ int (cl(A)) ⊆ A.

All the above given definitions are different and independent. The classes ofpre – open , semi – open , α – open , semi – pre – open and b – open sets ofa space (X,T) are usually denoted by PO (X,T) , SO (X,T) , T^{α}, SPO (X,T) and BO (X,T) respectively.

DEFINITION (1.3) : b – Regular Spaces :
A topological spaces (X,T) is said to be a b – regular space if for every b – closed subset F of X , and for every point $x \in X$ in the manner that $x \notin F$, there exist disjoint b – open sets $G,H \subseteq X$ s.t. $F \subseteq G$ and $x \in H$.

or

For every b – closed subset F of X and for each $x \in F^C$, there exist disjoint b-open sets containing F and x separately.

In other words a topological space (X,T) is a b – regular space if given a point $x \in X$ and a b – closed set $F \subseteq X$ such that $x \notin F$ then there exist b – open sets $G,H \subseteq X$ such that $F \subseteq G$, $x \in H$ and $G \cap H = \phi$.

DEFINITION (1.4) : b – T_3 Space :
A topological space (X,T) is defined to be a b- T_3 space if it is both b – regular and b- T_1 space.

EXAMPLES OF b – Regular space (1.5) :

(i) Every discrete space is b – regular.
Let x = {a,b}
T = { ϕ, {a}, {b}, x}
Here T is a discrete topology
T^c = { ϕ, { a] , { b } , x }
BO (X, T) = { ϕ, { a] , { b } , x}
BC (X , T) = { ϕ, { a] , { b } , x}
Here (X,T) is b – regular.

(ii) Every indiscrete space is b- regular.Let X be finite non – empty set such that

X= {a, b, c}
T = { ϕ, x}

T^c = { ϕ, x }
BO (X ,T) = { ϕ, { a] , { b } , {c }, {a, b} , {b, c }, {c,a} , x} = P (x)
BC(X ,T) = { ϕ, { a] , { b } , {c }, {a, b} , {b, c }, {c,a} , x}
Here , (X,T) is b – regular.

Theorem (1.6)– Every b – regular and T_1- space is a b – T_3 space.
Proof: Let (X , T)be a T_1 space.
Let x , y ∈X ; where x≠ y.
Then since X is a T_1– space,{x} is a closed set and in turns{x} is a b – closed set.
Hence , (X,T) is a b – T_1 space.
Since , a topological space (X,T) is a b- T_1 space iff every singleton set $\{x\} \subseteq X$ is b – closed.
[by a theorem].
This means that (X,T)being b- regular and b- T_1 space is a b – T_3 space.
Hence , the theorem.

Theorem (1.7) – If (X,T) is a topological space then the following statements are equivalent :
(i) (X,T) is b – regular.
(ii) For every point x ∈X and each b- closed set N containing x , there exists a b – closed set M containing x s.t. $M \subseteq N$.
(iii) every b – closed subset G of X is the intersection ofall super b – closed sets of G.
CHARACTERIZATION THEOREM

Theorem (1.8) - A topological space (X,T) is b- regular iff $\forall$ x $\in$X and every b – open set N containing x , there exists a b- open set M containing x s.t. b- cl(M)$\subseteq$0.

PROOF: Necessary Condition:
Let (X,T) be a b- regular space.
Let O be a b – open set containing x $\in$ X.
Since , (X,T) is b – regular hence $\ni$ b- open sets L,M s.t. $O^c \subseteq L$,
x $\in$M and L$\cap$M $\Rightarrow \phi$, since O^c is b- closed and x $\notin O^c$.
Again , L$\cap$M $\Rightarrow$M $\subseteq L^c$
$\Rightarrow$b cl (M) $\subseteq$b cl (L^C) = L^C
Since, L^c is b – closed.

Also, $O^c \subseteq L \Rightarrow L^c \subseteq O$.
So, that
b cl (M) $\subseteq L^c$ and $L^c \subseteq$ O
$\Rightarrow$ bcl (M) $\subseteq$ O Hence , Proved.

SUFFICIENT CONDITION:
Let the prescribed codition hold good and given as :
$\forall$ x $\in$X and every b – open set N containing x there exists a b-open set M containing x s.t. b cl (M) $\subseteq$N.
Let P be a b- closed set s.t. x $\notin$ P then x $\notin$ P $\Rightarrow$ x $\in P^c$ which is b – open.

Now , x $\in P^c \Rightarrow \ni$ a b – open set M
s.t. x $\in$M and b cl (M) $\subseteq P^c$.
[According to the prescribed condition]
$\Rightarrow$ P $\subseteq$ { b cl (M)$\}^c$.
Thus , x $\in$M $\wedge$ P $\subseteq$\{ b cl (M)$\}^c$
Which is b – open .

Also ,
M $\cap$ { b cl (M)$\}^c \subseteq$ [b cl (M)] $\cap$ [b cl (M)$]^c$
or , M $\cap$ { b cl (M)$\}^c \subseteq \phi$.
Since , ϕ is a subset of every set , hence M $\cap$ { b cl (M)$\}^c = \phi$.
Let, { b cl (M)$\}^c$ = G , then we obtain that , as x is arbitrary , for every x $\in$X and every b – closed set P for which x$\notin$P , there exist b – open sets M & G such that P $\subseteq$G and x$\in$M , where M$\cap$G $\Rightarrow \phi$. $\Rightarrow$ (X,T) is b – regular.
Hence , the sufficient codition.

THEROM (1.9): The property of a space being b – regular is a topological invarient under b – homomorphism .

PROOF: Let (X,T_1) be a b – regular space and (Y,T_2) is a b – regular space and (Y,T_2) is a b – homomorphic image of (X,T_1).
We have to show that (Y,T_2) is also b – regular.

Now , if H be a T_2 – b closed subset of Y and y $\in$Y s.t. y $\notin$ H , then f being one – one onto , $\ni$ x $\in$X s.t. f(x) = y $\Rightarrow f^{-1}$ (y) = x (1)

Also f : $T_1 \rightarrow T_2$is b – irresolute $\Rightarrow f^{-1}$ (H) is T_1 – b – closed

and y $\notin$ H $\Rightarrow$ f^{-1} (y) $\notin f^{-1}$(H)

$\Rightarrow$ x $\notin f^{-1}$(H) by (1)

$\Rightarrow f^{-1}(H)$ is T_1 –b closed and $x \in X$ s.t. $x \notin f^{-1}(H)$

$\Rightarrow$(X ,T) being b – regular , ∋ T_1 – b open sets N, M s.t. x∈N , f^{-1} (H)⊆M and N∩M $\Rightarrow \phi$.

But $X \in N \Rightarrow f(X) \in f(N) \Rightarrow y \in f(N)$ by (1),

$f^{-1}(H) \subseteq M \Rightarrow f(f^{-1}(H)) \subseteq f(M)$,

$\Rightarrow H \subseteq f(M)$

and N∩M $\Rightarrow \phi \Rightarrow f(N \cap M) = f(\phi)$

$\Rightarrow f(N) \cap f(M) = \phi$, since f is one – one and $f(\phi) = \phi$

Also , f is an M - b - open map

$\Rightarrow f(N) = N_1$ and $f(M) = N_2$ (say) are T_2 - b - open.

Conclusively , ∋ T_2 - b – open sets N_1 and M_1 s.t. y∈N_1 , H ⊆M and $N_1 \cap M_1 = \phi$.

It follows that (Y,T_2) is also a b – regular space and hence b – regularity is a topological invariant.

Hence , the theorem.

DEFINITION (1.10) :Completely b – Regular Spaces

A topological space (X,T) is said to be a completely b – regular space iff ∀ b – closed subset F of X and every point x of F^c there is a b – continuous function f on X into the subspace [0,1] of real line ,such that f (x) = 0 and f (y) = 1 ∀ y∈F .

In other words , a topological space (X,T) is a completely b – regular space if given a closed set F⊆X and a point x ∈X s.t. x ∉F , ∃a cotinuous function

$f: X \to [0,1]$ with $f(x) = 0$, $f(F) = \{1\}$.

DEFINITION (1.11):b - Tychonoff space or b – $T_{3/2}$ space

A Completely b – regular as well as b – T_1 space is said to be a b – Tychonoff space.

A b – Tychonoff space is known as b – $T_{3/2}$ space.

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