
INVENTORY MODEL FOR DECAYING ITEMS WITH TIME DEPENDENT DEMAND, LOST SALES AND TRADE CREDITS

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ABSTRACT

Each business enterprises play a imperative position in the improvement of the country. The determination of every business enterprise is to earn as much profit as probable while providing an adequate amenity level to its customers. The emphasis on customer service has gradually changed over the years and suppliers are now becoming really fascinated in customer service instead of just talking about it. Keeping customers satisfied means that they have the correct level of anticipations of supplies and that they are cheerful with their purchase so that there is potential for replication business and inclusive sales.

KEY WORDS : Customers, business, anticipations

INTRODUCTION

Every course of action flourishes on a customer policy, which best suits, its needs. Some people proffer discounts as part of their course of action, while some proffer free gifts with their products. The policies may be dissimilar, but the basic prerequisite is the same-to fix the customer. The trend in this field has also seen a drastic change with time as time has affected the business senses of organizations to a great extent. To keep a stride with the changing trends, vendors sometimes offer concessions or credit limits to their buyer. This means that the customer can take the goods home without having to pay for them instantly. Once the vendor provides a credit period policy, a buyer will decide his price, ordering and payment policies accordingly. During the period before the account has to be settled, the retailer can sell the product and continue to accumulate revenue and earn interest. The retailer can pay the vendor either at the end of the credit period or later on the incurring interest of the unpaid balance for the overdue period. Permissible delay period is a common attribute in practice. But with it we have to stick to a single time limit after which retailer will pay off. Here, we have considered that the supplier offers progressive credit period to settle the account by which supplier offers retailer two time period limits. If the retailer settles the outstanding amount by first time limit, the supplier does not charge any interest. If the retailer pays after the first time limit but before second period offered by the supplier, then the supplier charges the retailer on the un-paid balance. If the retailer settles the account after second time limit, then he will have to pay at interest rate more than to first limit on the un-paid balance. This way is beneficial for both parties. Supplier will get more interest from retailer and retailer will get more time and takes more advantage of credit policy. Such a credit limit usually influences the buyer into buying more, since it, in a way, reduces the purchasing cost for him. This pays off to the supplier as he gets the benefit of more sales, while the payment of the sale is sure to reach him within the stipulated frame of time.

REVIEW OF LITERATURE

Donaldson (1977) developed an optimal algorithm for solving classical no-shortage inventory model analytically with linear trend in demand over fixed time horizon. Goyal (1985) discussed such a situation in his paper. He derived a mathematical model for obtaining the EOQ for an item for which the supplier permits a fixed delay in settling the amount owed to him. A power demand pattern inventory model for deteriorating items was discussed by Dutta and

Pal (1988). Dave, U. (1989) proposed a deterministic lot-size inventory model with shortages and time dependent demand. Mandal and Phaujdar (1989) extended Goyal (1985) to incorporate shortages and considered the interest earned from sale revenues. Goswami and Chaudhuri (1991) discussed different types of inventory models with linear trend in demand. Hariga (1995) studied the effects of inflation and time value of money on an inventory model with time-dependent demand rate and shortages. Hwang and Shinn (1997) studied effects of permissible delay in payments on retailer's pricing and lot sizing policy for exponentially deteriorating products. Wang et al. (2000) analyzed supply chain models for perishable products under inflation and permissible delay in payment. Khanra and Chaudhuri (2003) discussed an order level decaying inventory model with such time dependent quadratic demand. The concept of such kind of variable demand rates is a realistic feature. Chung and Huang (2003) studied the optimal cycle time for EPQ inventory model under permissible delay in payments. Balkhi and Benkherouf (2014) developed an inventory model for deteriorating items with stock dependent and time varying demand rates over a finite planning horizon. Chung, Goyal and Huang, Y.F. (2005) considered the optimal inventory policies under permissible delay in payments depending on the ordering quantity. Song and Cai (2015) has been taken on optimal payment time for a retailer under permitted delay of payment by the wholesaler. Chen and Kang (2007) extended an integrated vendor-buyer cooperative inventory models with variant permissible delay in payments. Singh (2008) developed perishable inventory model with quadratic demand, partial backlogging and permissible delay in payments. Roy, A. (2014) introduced a deterministic inventory model for deteriorating items with price dependent demand and time varying holding cost. Singh, S.R. and Singh, S. (2008) developed a production inventory model for items with the effect of permissible delay in payments. Singh, S.R. and Singh, S. (2009) discussed the profit maximizing inventory model having exponentially increasing demand under trade credit.

ASSUMPTIONS

- Demand rate is an exponentially increasing function of time.
- Production rate is demand dependent.
- Deterioration rate is taken as constant.
- Shortages are allowed with partial backlogging.
- Backlogging rate is taken as exponential decreasing function of time.
- If the retailer pays by M, then the supplier does not charge to the retailer. If the retailer pays after M and before N ($N > M$), he can keep the difference in the unit sale price and unit purchase price in an interest bearing account at the rate of I_0 /unit/year. During $[M, N]$, the supplier charges the retailer an interest rate of I_{c1} /unit/year on unpaid balance. If the retailer pays after N, then supplier charges the retailer an interest rate of I_{c2} /unit/year ($I_{c1} > I_{c2}$) on unpaid balance.

NOTATIONS

- s = the selling price / unit.
- $D(t) = \lambda_0 e^{\alpha t}$, where $0 \leq \alpha \leq 1$ is a constant and λ_0 is the initial demand rate.
- KD = the production rate per year, where $K > 1$
- M = the first offered credit period in settling the account without any charges.
- N = the second permissible credit period in settling the account with interest charge I_{c2} on unpaid balance and $N > M$.
- I_{c1} = the interest charged per \$ in stock per year by the supplier when retailer pays during $[M, N]$.
- I_{c2} = the interest charged per \$ in stock per year by the supplier when retailer pays during $[N, T]$. ($I_{c1} > I_{c2}$)
- I_e = the interest earned / \$ / year.
- IE = the interest earned / time unit.
- IC = the interest charged / time unit.
- C = the unit purchase cost, with $C < s$.
- T = the replenishment cycle.
- $(C_{vh} + \phi t)$ = holding cost per unit time for vendor.

- $(C_{bh} + \phi t)$ = holding cost per unit time for buyer.
- C_{vs} = the setup cost for each production cycle for vendor.
- C_{bs} = the setup cost per order for buyer.
- S_b = shortage cost per unit time for buyer.
- SL_b = Lost sale cost per unit time for buyer.
- $e^{-\delta t}$ = Backlogging rate for buyer.
- C_v = the unit cost for vendor.
- C_b = the unit purchase cost for buyer.
- VC = the cost of vendor per unit time.
- BC = the cost of buyer per unit time.
- $TC(T)$ = total cost of an inventory system / time unit.

MODEL FORMULATION AND SOLUTION

In this model, vendor buyer inventory system for decaying items with exponential demand rate has been developed. The actual vendor's average inventory level in the integrated two-echelon inventory model is difference between the vendor's total average inventory level and the buyer's average inventory level. Since the inventory level is depleted due to a constant deterioration rate of the on-hand stock, the buyer's inventory level is represented by the following differential equations:

$$I_b'(t) + \theta I_b(t) = -\lambda_0 e^{\alpha t}, \quad 0 \leq t \leq t_1 \quad \dots(1.1)$$

$$I_b'(t) = -e^{-\delta t} \lambda_0 e^{\alpha t}, \quad t_1 \leq t \leq T \quad \dots(1.2)$$

The vendor's total inventory system consisting of production period and non-production period can be described as follows:

$$I_{v1}'(t) + \theta I_{v1}(t) = (K-1)\lambda_0 e^{\alpha t}, \quad 0 \leq t \leq T_1 \quad \dots(1.3)$$

$$I_{v2}'(t) + \theta I_{v2}(t) = -\lambda_0 e^{\alpha t}, \quad 0 \leq t \leq T_2 \quad \dots(1.4)$$

The boundary conditions are

$$I_{v1}(t) = 0, \quad t = 0 \quad \dots(1.5)$$

$$I_{v2}(t) = 0, \quad t = T_2 \quad \dots(1.6)$$

$$I_b(t) = I_0, \quad t = 0 \quad \dots(1.7)$$

$$I_b(t) = 0, \quad t = t_1 \quad \dots(1.8)$$

$$I_{v1}(T_1) = I_{v2}(0) \quad \dots(1.9)$$

and

$$T = \frac{T_2}{n} \quad \dots(1.10)$$

The solutions of the above differential equations obtained are

$$I_b(t) = \frac{\lambda_0}{\alpha + \theta} [e^{-\theta t} - e^{\alpha t}] + I_0 e^{-\theta t}, \quad 0 \leq t \leq t_1 \quad \dots(1.11)$$

$$I_b(t) = \frac{\lambda_0}{(\alpha - \delta)} [e^{(\alpha - \delta)t_1} - e^{(\alpha - \delta)t}], \quad t_1 \leq t \leq T \quad \dots(1.12)$$

$$I_{v1}(t) = \frac{(K-1)\lambda_0}{\alpha + \theta} [e^{\alpha t} - e^{-\theta t}], \quad 0 \leq t \leq T_1 \quad \dots(1.13)$$

$$I_{v2}(t) = \frac{\lambda_0}{\alpha + \theta} \left[\frac{e^{(\theta + \alpha)T_2} - e^{(\theta + \alpha)t}}{e^{\theta t}} \right], \quad 0 \leq t \leq T_2 \quad \dots(1.14)$$

Using the condition that we obtain :

$$I_0 = \frac{\lambda_0}{\alpha + \theta} \left[e^{(\alpha + \theta)t_1} - 1 \right] \quad \dots(1.15)$$

If the product of the deterioration rate and the replenishment interval is much smaller than one, the buyer's and the vendor's actual average inventory level, \bar{I}_b and \bar{I}_v , are

$$\bar{I}_b = \frac{(C_{bh} + \phi t)}{T} \int_0^{t_1} I_b(t) dt \quad \dots(1.16)$$

and

$$\bar{I}_v = \frac{(C_{vh} + \phi t)}{T_2} \left[\int_0^{T_1} I_{v1}(t) dt + \int_0^{T_2} I_{v2}(t) dt \right] - \bar{I}_b \quad \dots(1.17)$$

respectively

The annual total holding cost for the buyer and the vendor are

$$HC_b = C_b \bar{I}_b \quad \dots(1.18)$$

and

$$HC_v = C_v \bar{I}_v \quad \dots(1.19)$$

Respectively

The annual deterioration cost for the buyer and the vendor are

$$DC_b = \frac{C_b \theta}{T} \int_0^{t_1} I_b(t) dt \quad \dots(1.20)$$

and

$$DC_v = \frac{C_v \theta}{T_2} \left[\int_0^{T_1} I_{v1}(t) dt + \int_0^{T_2} I_{v2}(t) dt \right] - \bar{I}_b \quad \dots(1.21)$$

respectively

The annual set-up cost for the buyer and the vendor are

$$OC_b = \frac{C_{bs}}{T} \quad \dots(1.22)$$

and

$$OC_v = \frac{C_{vs}}{T_2} \quad \dots(1.23)$$

respectively

The annual shortage cost for the buyer is

$$SC_b = \frac{S_b}{T} \int_{t_1}^T (-I_b(t)) dt \quad \dots(1.24)$$

The annual lost sale cost for the buyer is

$$LSC_b = \frac{S_{Lb}}{T} \int_{t_1}^T (1 - e^{-\delta t}) \lambda_0 e^{\alpha t} dt \quad \dots(1.25)$$

The different costs associated with the system are set-up costs, holding costs, deterioration cost and shortage cost. Our aim is to minimize the total cost.

From (1.9), one can derive the following condition:

$$\frac{(K-1)\lambda_0}{\alpha + \theta} \left[e^{\alpha T_1} - e^{-\theta T_1} \right] = \frac{\lambda_0}{\alpha + \theta} \left[e^{(\alpha + \theta)T_2} - 1 \right] \quad \dots(1.26)$$

By Taylor's series expansion, (26) is derived as

$$T_1 = \frac{1}{K-1} T_2 \left[1 + \frac{(\alpha + \theta)}{2} T_2 \right] \quad \dots(1.27)$$

CONCLUSION

This paper deals inventory model for decaying items with exponential demand under progressive credit period. The outcome of this chapter shows that the retailer can reduce total annual relevant inventory cost when the supplier provides a permissible delay in payments. This model well suits to situations where shortages are allowed. Here the allowed shortages are partially backlogged with time dependent backlogging rate.

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