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EFFECT OF DETERIORATING INVENTORY MODEL FOR SHORTAGES AND TRAPEZOIDAL TYPE DEMAND RATE

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ABSTARCT

In this paper, It is important to control and maintain the inventories of deteriorating items for the modern corporation. We will discuss two models: one is without shortage, and the other is with shortage. We obtain the optimal solutions in this paper; we assume that the inventory objective is to minimize the total cost per unit time of the system.

KEYWORDS: Inventories, shortage

INTRODUCTION

The effect of deterioration is very important in many inventory systems. Most of the literature assumes that a constant proportion of items will deteriorate per time-unit while they are in storage. Ghare and Schrader were the first proponents for developing a model for an exponentially decaying inventory, to consider continuously decaying inventory for a constant demand [1]. Covert and Philip used a variable deterioration rate of two-parameter Weibull distribution to formulate the model with assumptions of a constant demand rate and no shortages [2]. Shah and Jaiswal presented an order-level inventory model for deteriorating items with a constant rate of deterioration [3]. Dave and Patel first considered the inventory model for deteriorating items with time-varying demand [4]. They considered a linear increasing demand rate over a finite horizon and a constant deterioration rate. Chang and Dye developed an EOQ model for deteriorating items with time-varying demand and partial backlogging [5]. Skouri and Papachristos presented a continuous review inventory model, with deteriorating items, time-varying demand , linear replenishment cost, partially time-varying backlogging [6]. Other researchers, there are many literatures that propose and evaluate the algorithms [7], [8], [9], [10], [11].

In the classical inventory model, the demand rate is assumed to be a constant. In reality, the demand for physical goods may be time-dependent, stock-dependent and price dependent. Hill first considered the inventory models for increasing demand followed by a constant demand [12]. M and al and Pal extended the inventory

model with ramp type demand for deterioration items and allowing shortage [13]. Chen, Ouyang and Teng considered an EOQ model with ramp type demand rate and time dependent deterioration rate [14]. P and a, Senapati and Basu developed optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand [15]. Other researchers, there are many literatures that propose and evaluate the algorithms [16], [17], [18], [19], [20], [21], [22], [23], [24][25].

ASSUMPTIONS:

1.The replenishment rate is infinite, thus replenishment rate is instantaneous.

2.The demand rate, D(t) which is positive and consecutive, is assumed to be a trapezoidal type function of time that is:

$$D(t)=\begin{cases} a_1+b_1t, t\le\mu_1 \\ D_0, \mu_1\le t\le\mu_2 \\ a_2-b_2t, \mu_2\le t\le T\le\dfrac{a_2}{b_2} \end{cases}$$

Where μ_1 is time point changing from the increasing linearly demand to constant demand, and μ_2 is time point changing from the constant demand to the decreasing linearly demand.

3. The length of each ordering cycle is fixed.
4. Deterioration rate is taken as time dependant.
5. Shortages occur and partially backlogged.
6. Backlogging rate is exponential decreasing function of time.

NOTATIONS:

I(t)	inventory level at any time t
T	the fixed length of each ordering cycle
Kt	Time dependent deterioration rate and K is a constant
t_1	the time when the inventory level reaches zero
A_0	fixed ordering cost per order
c_1	the cost of each deteriorated item
c_2	inventory holding cost per unit per unit of time
c_3	shortage cost per unit per unit of time
S	maximum inventory level
Q	ordering quantity per cycle
μ_1	time point changing from the increasing linearly demand to constant demand
μ_2	time point changing from the constant demand to the decreasing linearly demand
$e^{-\delta t}$	waiting time during shortages up-to next replenishment

MATHEMATICAL FORMULATION:

We considered an order level inventory model with trapezoidal type demand rate. Replenishment occurs at time t = 0 when the inventory level attains its maximum. From t = 0 to t_1, the inventory level reduces due to demand and deterioration. At t_1, the inventory level achieves zero, then shortage is allowed to occur during the time interval (t_1, T) and all of the demand during the shortage period is partially backlogged due to impatience of customer.

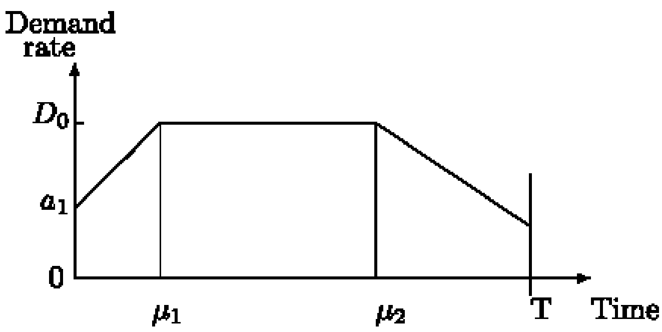


Fig. 1. A trapezoidal type function of the demand rate.

The differential equations governing the transition of the system are given by:

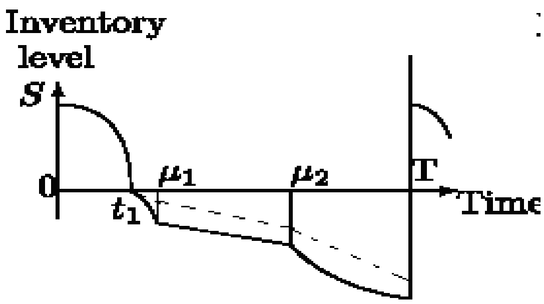
$$\frac{dI(t)}{dt}=-KtI(t)-D(t) \quad 0\le t\le t_1 \qquad \dots (1)$$

$$\frac{dI(t)}{dt} = -e^{-\delta t} D(t) \quad t_1 \le t \le T \qquad \text{... (2)}$$

With boundary condition $I(t_1)=0$

Now we consider three possible cases based on the values of t_1, μ_1 and μ_2. These three cases are shown as follows:

Case-1: When $0 \le t_1 \le \mu_1$



Case 1. $0 < t_1 \le \mu_1$

Due to combined effect of demand and deterioration the inventory level gradually diminishes during the period $[0, t_1]$ and ultimately falls to zero at time t_1. The differential equations are given by:

$$\frac{dI(t)}{dt} = -KtI(t) - (a_1 + b_1 t) \quad 0 \le t \le t_1 \qquad \text{... (3)}$$

$$\frac{dI(t)}{dt} = -(a_1 + b_1 t)e^{-\delta t} \quad t_1 \le t \le \mu_1 \qquad \text{... (4)}$$

$$\frac{dI(t)}{dt} = -D_0 e^{-\delta t} \quad \mu_1 \le t \le \mu_2 \qquad \text{.. (5)}$$

$$\frac{dI(t)}{dt} = -(a_2 - b_2 t)e^{-\delta t} \quad \mu_2 \le t \le T \qquad \text{.. (6)}$$

With boundary condition $I(t_1) = 0$

The solutions of these equations are given by:-

$$I(t) = [a_1(t_1 - t) + \frac{b_1}{2}(t_1^2 - t^2) + \frac{a_1 k}{6}(t_1^3 - t^3) + \frac{b_1 k}{8}(t_1^4 - t^4)]e^{-\frac{kt^2}{2}} \quad 0 \le t \le t_1 \qquad \text{....(7)}$$

$$I(t) = (a_1 + b_1 t)\frac{e^{-\delta t}}{\delta} - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta} + \frac{b_1}{\delta^2}(e^{-\delta t} - e^{-\delta t_1}) \quad t_1 \le t \le \mu_1 \qquad \text{.... (8)}$$

$$I(t) = \frac{D_0}{\delta}(e^{-\delta t} - e^{-\delta \mu_1}) + (a_1 + b_1 \mu_1)\frac{e^{-\delta \mu_1}}{\delta} - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta} + \frac{b_1}{\delta^2}(e^{-\delta \mu_1} - e^{-\delta t_1}) \quad \mu_1 \le t \le \mu_2 \qquad \text{... (9)}$$

$$I(t) = (a_2 - b_2 t)\frac{e^{-\delta t}}{\delta} - (a_2 - b_2 T)\frac{e^{-\delta t}}{\delta} + \frac{b_2}{\delta^2}(e^{-\delta T} - e^{-\delta t}) \quad \mu_2 \le t \le T \qquad \text{... (10)}$$

The inventory level at the beginning is given by:-

$$I(0) = [a_1 t_1 + \frac{b_1}{2}t_1^2 + \frac{a_1 k}{6}t_1^3 + \frac{b_1 k}{8}t_1^4] = S \qquad \text{... (11)}$$

The total no. of items which perish in the inventory $[0, t_1]$, say D_T, is:-

$$D_T = S - \int_0^{t_1} D(t).dt$$

So the cost of deterioration is given by:-

$$\text{Det. Cost} = (\frac{a_1 k}{6}t_1^3 + \frac{b_1 k}{8}t_1^4)c_1 \qquad \text{... (12)}$$

The total number of inventory carried during the interval $[0, t_1]$:

$$H_T = c_2 \int_0^{t_1} I(t)dt$$

$$= c_2 \left(a_1 \frac{t_1^2}{2} + b_1 \frac{t_1^3}{3} + \frac{a_1 k}{12} t_1^4 + \frac{b_1 k}{15} t_1^5\right) \quad \text{.... (13)}$$

The total shortage quantity during the interval $[t_1, T]$, say B_T, is:-

$$B_T = -\int_{t_1}^{\mu_1} I(t).dt - \int_{\mu_1}^{\mu_2} I(t).dt - \int_{\mu_2}^{T} I(t).dt$$

$$-\{(a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{-\delta^2} - \frac{b_1 e^{-\delta\mu_1}}{\delta^3} - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta}\mu_1 - \frac{b_1}{\delta^2}(\frac{e^{-\delta\mu_1}}{\delta} + \mu_1 e^{-\delta t_1}) +$$

$$(a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta^2} + \frac{b_1 e^{-\delta t_1}}{\delta^3} + (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta} t_1 + \frac{b_1}{\delta^2}(\frac{e^{-\delta t_1}}{\delta} + t_1 e^{-\delta t_1})\} -$$

$$B_T = \{\frac{D_0}{\delta}(\frac{e^{-\delta\mu_2}}{-\delta} - \mu_2 e^{-\delta\mu_1}) + (a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{\delta}\mu_2 - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta}\mu_2 + \frac{b_1}{\delta^2}(e^{-\delta\mu_1} - e^{-\delta t_1})\mu_2 +$$

$$\frac{D_0}{\delta}(\frac{e^{-\delta\mu_1}}{\delta} + \mu_1 e^{-\delta\mu_1}) - (a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{\delta}\mu_1 + (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta}\mu_1 - \frac{b_1}{\delta^2}(e^{-\delta\mu_1} - e^{-\delta t_1})\mu_1\} \quad \text{... (14)}$$

$$-\{(a_2 - b_2 T)\frac{e^{-\delta T}}{-\delta^2} + \frac{b_2 e^{-\delta T}}{\delta^3} - (a_2 - b_2 T)\frac{e^{-\delta T}}{\delta} T + \frac{b_2}{\delta^2}(e^{-\delta T} T + \frac{e^{-\delta T}}{\delta}) +$$

$$(a_2 - b_2\mu_2)\frac{e^{-\delta\mu_2}}{\delta^2} - \frac{b_2 e^{-\delta\mu_2}}{\delta^3} + (a_2 - b_2 T)\frac{e^{-\delta T}}{\delta}\mu_2 - \frac{b_2}{\delta^2}(e^{-\delta T}\mu_2 + \frac{e^{-\delta\mu_2}}{\delta})\}$$

Shortage Cost = $C_3 B_T$ ... (15)

Then, the average total cost per unit time under the condition $t_1 \le \mu_1$ can be given by:

$$C_1(t_1, T) = \frac{1}{T}[A_0 + c_1 D_T + c_2 H_T + c_3 B_T] \quad \text{... (16)}$$

The necessary conditions for the total relevant cost per unit time of the equation (16) is to be minimized is

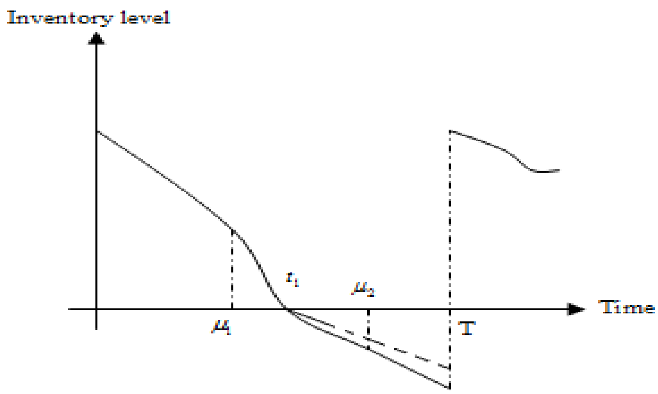
$$\frac{\partial C_1(t_1, T)}{dt_1} = 0 \text{ and } \frac{\partial C_1(t_1, T)}{dT} = 0$$

$$\left.\frac{\partial^2 C_1(t_1, T)}{\partial t_1^2}\right| > 0, \left.\frac{\partial^2 C_1(t_1, T)}{\partial T^2}\right| > 0$$

$$\left.\left(\frac{\partial^2 C_1(t_1, T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C_1(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 C_1(t_1, T)}{\partial t_1 \partial T}\right)\right| > 0 \quad \text{... (17)}$$

Case 2: $\mu_1 \le t_1 \le \mu_2$

The differential equations governing the transition of the system are given by:-



Case 2. $\mu_1 \le t_1 \le \mu_2$

$$\frac{dI(t)}{dt} = -KtI(t) - (a_1 + b_1 t) \quad 0 \le t \le \mu_1 \qquad \ldots (18)$$

$$\frac{dI(t)}{dt} = -KtI(t) - D_0 \quad \mu_1 \le t \le t_1 \qquad \ldots (19)$$

$$\frac{dI(t)}{dt} = -D_0 e^{-\delta t} \quad t_1 \le t \le \mu_2 \qquad \ldots (20)$$

$$\frac{dI(t)}{dt} = -(a_2 - b_2 t)e^{-\delta t} \quad \mu_2 \le t \le T \qquad \ldots (21)$$

With boundary condition I(t_1) = 0

The solutions of these equations are given by:-

$$I(t) = [a_1(\mu_1 - t) + \frac{a_1 k}{6}(\mu_1^3 - t^3) + \frac{b_1}{2}(\mu_1^2 - t^2) + \frac{b_1 k}{8}(\mu_1^4 - t^4) + D_0\{(t_1 - \mu_1) + \frac{k}{6}(t_1^3 - \mu_1^3)\}]e^{-\frac{kt^2}{2}} \quad 0 \le t \le \mu_1 \qquad \ldots (22)$$

$$I(t) = D_0\{(t_1 - t) + \frac{K}{6}(t_1^3 - t^3)\}e^{-\frac{Kt^2}{2}} \quad \mu_1 \le t \le t_1 \qquad \ldots (23)$$

$$I(t) = \frac{D_0}{\delta}(e^{-\delta t} - e^{-\delta t_1}) \quad t_1 \le t \le \mu_2 \qquad \ldots (24)$$

$$I(t) = (a_2 - b_2 t)\frac{e^{-\delta t}}{\delta} - (a_2 - b_2 T)\frac{e^{-\delta T}}{\delta} - \frac{b_2}{\delta^2}(e^{-\delta t} - e^{-\delta T}) \quad \mu_2 \le t \le T \qquad \ldots (25)$$

The beginning inventory level is:-

$$I(0) = S = \{a_1\mu_1 + \frac{a_1 K}{6}\mu_1^3 + \frac{b_1\mu_1^2}{2} + \frac{b_1 K\mu_1^4}{8} + D_0(t_1 + \frac{K}{6}t_1^3)\} \qquad \ldots (26)$$

The total no. of items which perish in the inventory $[0, t_1]$, say D_T, is:-

$$D_T = S - \int_0^{t_1} D(t).dt$$

So the cost of deterioration is given by:

$$\text{D.C.} = \{a_1\mu_1 + \frac{a_1 K}{6}\mu_1^3 + \frac{b_1\mu_1^2}{2} + \frac{b_1 K\mu_1^4}{8} + D_0(t_1 + \frac{K}{6}t_1^3) - (a_1\mu_1 + \frac{b_1\mu_1^2}{2}) - D_0(t_1 - \mu_1)\}c_1 \qquad \ldots (27)$$

The total number of inventory carried during the interval $[0, t_1]$: -

$$H_T = \int_0^{t_1} I(t)dt$$

$$= [a_1\frac{\mu_1^2}{2} + a_1\frac{K\mu_1^4}{8} + \frac{b_1}{3}\mu_1^3 + b_1\frac{K\mu_1^5}{10} + D_0\{\mu_1(t_1 - \mu_1) + \frac{K}{6}\{\mu_1(t_1^3 - \mu_1^3)\}$$

$$-\frac{a_1 K}{24}\mu_1^4 - \frac{b_1 K}{30}\mu_1^5 - D_0\frac{K}{2}\{\frac{\mu_1^3}{3}(t_1 - \mu_1)\} + D_0(\frac{t_1^2}{2} + \frac{K}{8}t_1^4) - \frac{D_0 K}{24}t_1^4 \qquad \ldots (28)$$

$$-D_0\{(t_1\mu_1 - \frac{\mu_1^2}{2}) + \frac{K}{6}(t_1^3\mu_1 - \frac{\mu_1^4}{4})\} + \frac{D_0 K}{2}(t_1\frac{\mu_1^3}{3} - \frac{\mu_1^4}{4})]$$

The total shortage quantity during the interval $[t_1, T]$, say B_T, is:-

$$B_T = -\int_{t_1}^{\mu_2} I(t).dt - \int_{\mu_2}^{T} I(t).dt$$

$$B_T =$$

$$[\frac{D_0}{\delta}(\frac{e^{-\delta\mu_2} - e^{-\delta t_1}}{\delta} + e^{-\delta t_1}(\mu_2 - t_1)) + (a_2 - b_2 T)\frac{e^{-\delta T}}{\delta^2} - (a_2 - b_2\mu_2)\frac{e^{-\delta\mu_2}}{\delta^2}$$

$$-2\frac{b_2}{\delta^3}(e^{-\delta T} - e^{-\delta\mu_2}) + (a_2 - b_2 T)(T - \mu_2)\frac{e^{-\delta T}}{\delta} - \frac{b_2}{\delta^2}e^{-\delta T}(T - \mu_2)] \qquad \ldots (29)$$

Then the average total cost per unit time for this case is given by:-

$$C_2(t_1) = \frac{1}{T}[A_0 + c_1 D_T + c_2 H_T + c_3 B_T] \qquad \ldots (30)$$

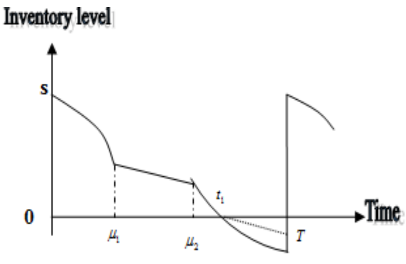
The necessary conditions for the total relevant cost per unit time of the equation (30) is to be minimized is

$$\frac{\partial C_2(t_1,T)}{dt_1}=0 \text{ and } \frac{\partial C_2(t_1,T)}{dT}=0$$

$$\left.\frac{\partial^2 C_2(t_1,T)}{\partial t_1^{\,2}}\right|>0, \left.\frac{\partial^2 C_2(t_1,T)}{\partial T^2}\right|>0$$

$$\left.\left(\frac{\partial^2 C_2(t_1,T)}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 C_2(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 C_2(t_1,T)}{\partial t_1 \partial T}\right)\right|>0 \quad \ldots (17)$$

Case-3: $\mu_2 \le t_1 \le T$



$\mu_2 \le t_1 \le T$

The differential equations governing the transition of the system are given by:-

$$\frac{dI(t)}{dt}=-KtI(t)-(a_1+b_1t) \quad 0\le t\le \mu_1 \quad \ldots (31)$$

$$\frac{dI(t)}{dt}=-KtI(t)-D_0 \quad \mu_1\le t\le \mu_2 \quad \ldots (32)$$

$$\frac{dI(t)}{dt}=-KtI(t)-(a_2-b_2t) \quad \mu_2\le t\le t_1 \quad \ldots (33)$$

$$\frac{dI(t)}{dt}=-(a_2-b_2t)e^{-\delta t} \quad t_1\le t\le T \quad \ldots (34)$$

Using boundary condition $I(t_1)=0$

Solution of these equation are given by-

$$I(t)=\{S-(a_1t+\frac{b_1}{2}t^2+\frac{a_1K}{6}t^3+\frac{b_1K}{8}t^4)\}e^{-\frac{Kt^2}{2}} \quad 0\le t\le \mu_1 \quad \ldots (35)$$

$$I(t)= \{D_0((\mu_2-t)+\frac{K}{6}(\mu_2^{\,3}-t^3))+a_2(t_1-\mu_2)+\frac{a_2K}{6}(t_1^{\,3}-\mu_2^{\,3})-\frac{b_2}{2}(t_1^{\,2}-\mu_2^{\,2})-\frac{b_2K}{8}(t_1^{\,4}-\mu_2^{\,4})\}e^{-\frac{Kt^2}{2}}$$

$$\mu_1\le t\le \mu_2 \quad \ldots (36)$$

$$I(t)=\{a_2(t_1-t)+\frac{a_2K}{6}(t_1^{\,3}-t^3)-\frac{b_2}{2}(t_1^{\,2}-t^2)-\frac{b_2K}{8}(t_1^{\,4}-t^4))\}e^{-\frac{Kt^2}{2}}\} \quad \mu_2\le t\le t_1 \quad \ldots (37)$$

$$I(t)=(a_2-b_2t)e^{-\frac{\delta t}{\delta}}-(a_2-b_2t_1)e^{-\frac{\delta t_1}{\delta}}-\frac{b_2}{\delta^2}(e^{-\delta t}-e^{-\delta t_1}) \quad t_1\le t\le T \quad \ldots (38)$$

$$S=D_0\{(\mu_2-\mu_1)+\frac{K}{6}(\mu_2^{\,3}-\mu_1^{\,3})\}+a_2(t_1-\mu_2)+\frac{a_2K}{6}(t_1^{\,3}-\mu_2^{\,3})$$
$$-\frac{b_2}{2}(t_1^{\,2}-\mu_2^{\,2})-\frac{b_2K}{8}(t_1^{\,4}-\mu_2^{\,4})\}+(a_1\mu_1+\frac{a_1K}{6}\mu_1^{\,3}+b_1\frac{\mu_1^{\,2}}{2}+\frac{b_1K}{8}\mu_1^{\,4}) \quad \ldots (39)$$

The total number of items which deteriorate in the interval [0, t_1] is

$$D_T=S-\int_0^{t_1}D(t)dt$$

$$D_T=S-\{(a_1\mu_1+b_1\frac{\mu_1^{\,2}}{2})-D_0(\mu_2-\mu_1)-(a_2t_1-b_2\frac{t_1^{\,2}}{2}-\mu_2a_2+b_2\frac{\mu_2^{\,2}}{2})\} \quad \ldots (40)$$

The total number of inventory carried during the interval [$0,t_1$] :

$$H_T=\int_0^{t_1}I(t)dt$$

$$H_T = \begin{array}{l} \{S\mu_1 - (a_1\frac{\mu_1^2}{2} + \frac{a_1K}{24}\mu_1^4 + \frac{b_1}{6}\mu_1^3 + \frac{b_1K}{40}\mu_1^5) - \frac{KS}{6}\mu_1^3 + \frac{K}{2}(a_1\frac{\mu_1^4}{4} + \frac{b_1}{10}\mu_1^5) + \{D_0(\frac{\mu_2^2}{2} + \frac{K}{8}\mu_2^4) + \\ a_2(t_1-\mu_2)(\mu_2-\mu_1) + \frac{a_2K}{6}(t_1^3-\mu_2^3)(\mu_2-\mu_1) - \frac{b_2}{2}(t_1^2-\mu_2^2)(\mu_2-\mu_1) - \frac{b_2K}{8}(t_1^4-\mu_2^4)(\mu_2-\mu_1) \\ -\frac{D_0k}{2}(\frac{\mu_2^4}{12} - \mu_2\frac{\mu_1^3}{3} + \frac{\mu_1^4}{4}) - \frac{a_2K}{6}(t_1-\mu_2)(\mu_2^3-\mu_1^3) + \frac{b_2K}{12}(t_1^2-\mu_2^2)(\mu_2^3-\mu_1^3) - \\ D_0\{(\mu_2\mu_1 - \frac{\mu_1^2}{2}) + \frac{K}{6}(\mu_2^3\mu_1 - \frac{\mu_1^4}{4})\}\} + \{a_2\frac{t_1^2}{2} - \frac{b_2}{3}t_1^3 + \frac{a_2K}{12}t_1^4 - \frac{b_2K}{15}t_1^5 - a_2(t_1\mu_2 - \frac{\mu_2^2}{2}) - \\ \frac{a_2K}{6}(t_1^3\mu_2 - \frac{\mu_2^4}{4}) + \frac{b_2}{2}(t_1^2\mu_2 - \frac{\mu_2^3}{3}) + \frac{b_2K}{8}(t_1^4\mu_2 - \frac{\mu_2^5}{5}) + \frac{K}{2}a_2(t_1\frac{\mu_2^3}{3} - \frac{\mu_2^4}{4}) - \frac{b_2K}{4}(t_1^2\frac{\mu_2^3}{3} - \frac{\mu_2^5}{5})\} \end{array} \quad \text{... (41)}$$

The total shortage quantity during the interval $[t_1, T]$ is:

$$B_T = -\int_{t_1}^{T} I(t).dt$$

$$B_T = \begin{array}{l} (a_2 - b_2t_1)\frac{e^{-\delta t_1}}{\delta}(T-t_1) - \frac{b_2}{\delta^3}(e^{-\delta T} - e^{-\delta t_1}) - \frac{b_2}{\delta^2}e^{-\delta t_1}(T-t_1) + (a_2 - b_2T)\frac{e^{-\delta T}}{\delta^2} - \\ (a_2 - b_2t_1)\frac{e^{-\delta t_1}}{\delta^2} - \frac{b_2}{\delta^3}(e^{-\delta T} - e^{-\delta t_1}) \end{array} \quad \text{... (42)}$$

Then the average total cost per unit time for this case is given by:-

$$C_3(t_1) = \frac{1}{T}[A_0 + c_1D_T + c_2H_T + c_3B_T] \quad \text{.... (43)}$$

The necessary conditions for the total relevant cost per unit time of the equation (30) is to be minimized is

$$\frac{\partial C_3(t_1,T)}{dt_1} = 0 \text{ and } \frac{\partial C_3(t_1,T)}{dT} = 0$$

$$\left.\frac{\partial^2 C_3(t_1,T)}{\partial t_1^2}\right| > 0, \left.\frac{\partial^2 C_3(t_1,T)}{\partial T^2}\right| > 0$$

$$\left.\left(\frac{\partial^2 C_3(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C_3(t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 C_3(t_1,T)}{\partial t_1 \partial T}\right)\right| > 0 \quad \text{... (44)}$$

Table 1: Variation in total cost with the variation in K

K	t_1	T.C
0.0004	1.38958	14557.7
0.0006	1.38963	14557.8
0.0008	1.38967	14557.8
0.001	1.38972	14557.9
0.0012	1.38977	14557.9
0.0014	1.38981	14558
0.0016	1.38986	14558
0.0018	1.3899	14558.1
0.002	1.38995	14558.1

Table 2: Variation in total cost with the variation in a_1

a_1	t_1	T.C.
100	1.19764	12409.3
110	1.24414	12832.8
120	1.28598	13259.8
130	1.32383	13689.9
140	1.35826	14122.7
150	1.38972	14557.9
160	1.41858	14995.1
170	1.44517	15434.1
180	1.46974	15874.7
190	1.49252	16316.8
200	1.5137	16760.1

Table 3: Variation in total cost with the variation in a_2

a_2	t_1	T.C.
200	1.38972	15757.9
210	1.38972	15517.9
220	1.38972	15277.9
230	1.38972	15037.9
240	1.38972	14797.9
250	1.38972	14557.9
260	1.38972	14317.9
270	1.38972	14077.87
280	1.38972	13837.9
290	1.38972	13597

Variation in total cost with the variation in b_1

b_1	t_1	T.C
2.5	1.66521	9445.65
3	1.60069	10454.9
3.5	1.54151	11471.6
4	1.48698	12494.8
4.5	1.43655	13523.7
5	1.38972	14557.9
5.5	1.60875	15515.6
6	1.83054	16302.8
6.5	2.05507	16899.1
7	2.28233	17282.8

Variation in total cost with the variation in b_2

b_2	**t_1**	**T.C**
5	1.38972	12515.2
5.5	1.38972	13025.9
6	1.38972	13536.5
6.5	1.38972	14047.2
7	1.38972	14557.9
7.5	1.38972	15068.5
8	1.38972	15579.2
8.5	1.38972	16089.9
9	1.38972	16600.5
9.5	1.38972	17111.2
10	1.38972	17621.9

Variation in total cost with the variation in μ_1

μ_1	**t_1**	**T.C**
2.5	0.33593	4364.31
3	0.541165	5790.94
3.5	0.749162	7524.75
4	0.959921	9564.56
4.5	1.17344	11909.3
5	1.38972	14557.9
5.5	1.60875	17509.3
6	1.83054	20762.7
6.5	2.05507	24317
7	2.28233	28171.6

Variation in total cost with the variation in μ_2

μ_2	t_1	T.C
6	1.68238	14525.2
6.5	1.64553	14722.8
7	1.60875	14865.2
7.5	1.57206	14952.3
8	1.53544	140184
8.5	1.49889	14960.5
9	1.46242	14881.6
9.5	1.42603	14747.4
10	1.38972	14557.9
10.5	1.35348	14313
11	1.31732	14012.7

Variation in total cost with the variation in c_2

c_2	t_1	T.C
0.3	1.37066	14528.4
0.35	1.35538	14535.7
0.4	1.38012	14543
0.45	1.3849	14550.4
0.5	1.38972	14557.9
0.55	1.39459	14565.4
0.6	1.39945	14572.9
0.65	1.40436	14580.5
0.7	1.40931	14588.2
0.75	1.4143	14595.9
0.8	1.41932	14603.7

CONCLUSION:

We have dealt with an EOQ inventory model for deteriorating items. It is assumed that the deterioration rate is time dependent. The nature of demand of seasonal and fashionable products is increasing-steady-decreasing and becomes asymptotic. For seasonal products like clothes, air conditions etc, demand of these items is very high at the starting of the season and become steady mid of the season and thereafter decreasing at the end of the season. The demand pattern assumed here is found to occur not only for all types of seasonal products but also for fashion apparel, computer chips of advanced computers, spare parts, etc.

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