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SOLUTION OF TSP WITH SIMULATED ANNEALING OF MANN

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ABSTRACT: Various problems of combinatorial optimization and permutation can be solved with neural network optimization Technique. Traveling salesman problem is one of the important examples of such category, which is characterized by its large number of iterating degree of freedom. There are various other solutions for this problem has been given in the past but mostly the methods are exact and heuristic. All the exact approaches can be considered for the theoretical interest. In this paper, we have proposed the solution of the problem by the Simulated Annealing of mean field approximation for describing the possible minimum length path that satisfies all the necessary constraints of the problem. The new energy function with the mean field approximation is also been proposed. The annealing schedule is also defined dynamically which is changing with the distance on each iteration of the processing. The algorithm of the whole process shows that this approach can generate the optimal solution.

KEYWORDS: TSP, Mean Field Approximation, Simulated Annealing, and Optimization.

1. INTRODUCTION

Most of the traditional problems of combinatorial permutation can be solved with the help of Artificial Neural Network [1], as it is well known that the ANN consists of various non-linear processing units [2]. These processing units can be interconnected in various ways or topologies [3]. The one form of this topology is in feedback manner. In this form, a set of processing units, connected to each processing unit except to itself. The output of each unit is feed as input to all other units. With each link connecting any two units, a weight is associated, which determines the amount of output, a unit feeds as input to the other units. The function of a feedback network with nonlinear units can be described in terms of the trajectory of the state of the network with time. By associating an energy function with the each state, the trajectory describes a traversal along the energy landscape. The minima of the energy landscape correspond to the stable states, which can be used to store the given input patterns. The numbers of patterns that can be stored in the network depends upon the number of units and the strength of the connecting links. The state of the network at successive instants of time i.e. the trajectory of the states is determined by the activation dynamics [4], for the network. Any pattern can be stored and recalled from such type of network [5]. During the process of the recalling the pattern, the network reaches to an equilibrium state [6], with the activation and synaptic dynamics. Associated with each output state is an energy [7], which depends on the network parameters like the weights and bias, besides the state of the network. The energy as a function of the state of the network corresponds to an energy landscape. One of the most prevalent uses of Neural Network is neural optimization which is a technique for solving a problem by casting it into a mathematical equation that, when either maximized or minimized, solves the problem without going into detailed dynamics of the concerned physical system. In other words, one of the most successful applications of the neural network principles is in solving optimization problem [8, 9]. There are many situations where a problem may be formulated as Minimization or Maximization of some cost function or objective function subject to constraints. It is possible to map such problem onto a feedback network, where the units and connection strengths are identified by comparing the cost function of the problem with the energy function of the network expressed in terms of the states values of the units and the connection strength. It has been demonstrated [10] that how highly interconnected network of simple analog processor can collectively compute good solutions to different optimization problems.

One of the most studied problems in the context of optimization using the method of neural networks is the Traveling Salesman Problem, where the objective is to find out the shortest route connecting all the cities to be visited by a salesman [11]. There are various solution of this problem has been given [12-20], there are various attempts have been made for finding the appropriate solution of Traveling Salesman Problem(TSP), in which randomized improvement heuristics is a popular one, was proposed by Junger, Reinelt and Rinaldi[12]. Some other approaches to solve extremely large TSPs (having tens to thousands or millions of variables) were proposed by Johnson, and Junger[13], Reinelt and Rinaldi[12], genetic algorithmic and neural net approaches had been proposed by Potvin[14,15], simulated annealing approach had been proposed by Aarts, et al[16], and tabu search approach had been proposed by Fiechter[17]. Performance guarantees for heuristics had been given by Johnson and Papadimitriou [18], the probability analysis of heuristics are given by Karp and Steele [19] and the development empirical testing of heuristics is reported by Golden and Stewart [20]. Another method for solving the problem was proposed by Behzad Kamgar-Parsi and Behrooz Kamgar-Parsi is based on analyzing dynamical stability of valid solutions, which yields relationships among search space for parameter values can't be arbitrary, thus the search space for parameter values becomes greatly restricted and therefore finding optimal values becomes much less tedious [21]. These solutions are exact and heuristic methods, but all the exact approaches are of considerable theoretical interest. The traditional method to solve such problems is gradient decent approaches of hill climbing and a stochastic simulated annealing. Hopfield and Tank proposed the solution of TSP, based on inherently parallel heuristic [2] and the Mean Field Annealing (MFA) algorithm [22, 23].

In this paper, we propose the Simulated Annealing of Mean field approximation method for describing the possible minimum path that satisfying all the necessary constraints on the problem. Energy function for any Hopfield type Feedback Neural Network can be represented that will satisfy all the imposed constraints of the problem. A global constraint in the form of distance of the traveled path (that is being selected randomly) can be selected for the Annealing schedule. The constraints can be reduced as per the schedule and correspondingly the new energy function is being estimated. It can be seen that the possible minimum energy function will represent the minimum distance path of the traveling salesman.

2. SIMULATED ANNEALING OF MFA FOR TSP

The traveling sales man problem is an well known example of the combinatorial optimization problem, which is characterized by its large number of interacting degrees of freedom. For a given number of cities (N) and their intercity distances, the objective is to determine a closed loop of the tour of cities, such that the total distance is minimized subjected to the constraints, that each city is visited only once and all the cities are covered in the tour. The Hopfield memory can be used to solve this problem. In this process the characteristic of interest is the rapid minimization of the energy function. To use the Hopfield memory for the application, we map the problem onto the Hopfield type network architecture. The first term is to develop a representation of the problem of the solution that fits an architecture having a single array of the processing elements (PE). We develop it by allowing a set of N PEs to represent the N possible positions for a given city in the sequence of the tour. The weight matrix format can be found from the city position. The output will be labeled as Vx_i, where the 'X', subscript refers to the city and the 'i', subscript refers to the position on the tour. To formulate the connection weight matrix, the energy function must be constructed that satisfying the following criteria:

a. Energy minima must favor states that have each city only once on the tour.
b. Energy minima must favor states that have each position on the tour only once.
c. Energy minima must favor states that include all N cities.
d. Energy minima must favor states with the shortest total distance.

Devoting the state of a processing unit of a Hopfield network as $Vx_i = 1$ indicates that the city X is to be visited at the i^{th} stage of the tour, the energy function [24] can be written as

$$E=\frac{A}{2}\sum_{X=1}^{N}\sum_{i=1}^{N}\sum_{j=1,j\neq 1}^{N}V_{X_i}V_{X_j}+\frac{B}{2}\sum_{i=1}^{N}\sum_{X=1}^{N}\sum_{Y=1,Y\neq X}^{N}V_{X_i}V_{Y_i}$$
$$+\frac{C}{2}(\sum_{X=1}^{N}\sum_{i=1}^{N}V_{X_i}-n)^2$$
$$+\frac{D}{2}\sum_{X=1}^{N}\sum_{Y=1,Y\neq X}^{N}\sum_{i=1}^{N}d_{xy}V_{X_i}(V_{Y,i+1}+V_{Y,i-1}) \qquad (2.1)$$

The Mean field Annealing algorithm [22, 23] also proposed a solution for this problem and with the MFA, the following energy function can be proposed [23] as,

$$E_{TSP}=\frac{d_{\max}}{2}\sum_{i}^{N}\sum_{j\neq i}^{N}\sum_{X}^{N}V_{X_i}V_{X_j}$$

$$+\frac{1}{2}\sum_{X}^{N}\sum_{Y\neq X}^{N}\sum_{i}^{N}d_{XY}V_{X_i}(V_{Y,i+1}+V_{Y,i-1}) \qquad (2.2)$$

Where d_{max} is real constant, which is slightly larger then the largest distance between the cities in the given TSP instance. So, in the equation (2.2), there is only two terms. The first term is regarding feasibility, which inhibits two cities from being in the same tour position. The second summation term is used for the minimization of the tour length. The term d_{max} is used for balancing of the summation terms.

The output Vx_i of a neuron (X, i) is interpreted as the probability of finding city X in tour position i. The mean field for a neuron (X, i) is defined according to the energy function given in equation (2.2) as,

$$E_{x_i}=d_{\max}\sum_{Y\neq X}^{N}V_{X_i}+\sum_{Y\neq X}^{N}d_{XY}(V_{Y,i+1}+V_{Y,i-1}) \qquad (2.3)$$

Initially all the neurons are arranged to the average value and the annealing schedule T with the d_{max}. The weights of the interconnections are initialized with the small random numbers. As per the Hopfield model the states of the any i^{th} neuron can be define as,

$$V_i(t+1)=f[\sum_{j\neq i}W_{ij}V_j(t)] \qquad (2.4)$$

As the dynamics given in equation (2.4) the network searches the stable state. This stable state may lead to a state corresponding to a local minimum of the energy function. In order to reach to the global minimum, i.e. the possible minimum path traveled by the salesman, by passing the local minima, we use the concept of stochastic updation of the unit in the activation dynamics of the network. In stochastic updation, the state of a unit is updated using the probabilistic updation, which is controlled by the annealing schedule constraint parameter ($T = d_{max}$). At the lower value of the constraint parameter, the stochastic update approaches the deterministic update, which is directed by the output function of the unit.

The probability distributions of the states at thermal equilibrium [14] can be written as,

$$P(V_{X_i})=\frac{1}{Z}e^{-E_{X_i}/T} \qquad (2.5)$$

Where Z is the partition function.

Initially the arbitrary city is selected and the energy function is constructed as given in equation (2.2). The Annealing schedule assigned with the maximum distance of the path i.e. to its higher value. So at higher T, many states are likely to be visited, irrespective of the energies of those states. Therefore for as per the Simulated Annealing Schedule, the value of T is gradually reduced, the output value of the states perturbs**.** This perturbation continues until the network settles to an stable state or equilibrium state. So, the network estimates the energy function for this state and compares it with the previous energy function by computing ΔE. If $\Delta E \leq 0$, we accept the solution with the highest probability i.e. 1, otherwise we accept it with the probability given in the equation (2.5). The state probabilities are computed by collecting the distribution of the states for a large number of cycles of updates of the states of the network at a given T. The cycles are repeated until the probabilities of the states do not change substantially for the different sets of cycles. On each iteration of this process, the solution is accepted with the probability 1, the Annealing Schedule parameter T is being assigned with the new d_{max} of the newly constructed energy function. Then every time on the acceptable solution with higher probability the constraints parameter changes with the newly found maximum distance. This newly found maximum distance would be less then the previously found maximum distance. So the Simulated Annealing process will continue with the new value of the constraint parameter. This process is continuing till the final value of the schedule. At this state the units of the network represents the state of equilibrium, which will represents the minimum energy function for the network. Thus the minimum energy function will represent the possible shortest path for the traveling salesman problem.

In order to speed up the process of the Simulated Annealing the Mean field approximation is used [25], in which the stochastic update of the binary units is replaced by deterministic analog states [26]. Thus the fluctuating activation value of each unit is replaced with its average value. The equation (2.5) can be express with this method as,

$$<V_i(t+1)>= f[<\sum_{j\neq i} W_{ij}V_j(t)>] = f\sum_{j\neq i} W_{ij} <V_j(t)> \qquad (2.6)$$

and from the stochastic updation with thermal equilibrium we have,

$$<V_i(t+1)>= \tanh[\frac{1}{T}\sum_{j\neq i}^{N} W_{ij} <V_j(t)>] \qquad (2.7)$$

This equation is solved iteratively starting with some arbitrary values $<V_i(0)>$ initially. Once the steady equilibrium values of $<V_i>$ have been obtained, the value of T is lowered. The next set of average states at thermal equilibrium is determined using the average state values at the previous thermal equilibrium condition of the initial value $<V_i(0)>$ in the equation (2.6) for iterative solution. The set equation of the Mean field approximation is a result of minimization of an effective energy defined as the function of T [27] as,

$$<V_i>= \tanh[-\frac{1}{T}\frac{\partial E(<V_i>)}{\partial <V_i>}] \qquad (2.8)$$

Where the effective energy E ($<V_i>$) is the expression for energy of the Hopfield model using averages for the state variables.

Now the constraint parameter T decreases as per the Annealing schedule of Mean field approximation, the state of the network perturbs with the stochastic asynchronous updation of the processing elements.

So, as the output value perturbs, the neurons produces the updated states. The states updation continues until the equation (2.6) satisfied.

Hence, for the stability conditions the energy function on each iteration of annealing schedule of Mean field approximation can be expressed as,

$$E_{TSP} = \frac{d_{\max}^{new}}{2}\sum_{i}^{N}\sum_{j\neq i}^{N}\sum_{X}^{N} <V_{X_i}><V_{X_j}> + \frac{1}{2}\sum_{X}^{N}\sum_{Y\neq X}^{N}\sum_{i}^{N} d_{XY} <V_{X_i}>(<V_{Y,i+1}+V_{Y,i-1}>) \qquad (2.9)$$

Hence, the energy difference ΔE can be computed as,

$$\Delta E = E_{TSP}^{new} - E_{TSP}^{old} = \frac{d_{\max}^{new}}{2}\sum_{i}^{N}\sum_{j\neq i}^{N} <V_{X_i}^{new}><V_{X_j}^{new}>$$
$$+ \frac{1}{2}\sum_{X}^{N}\sum_{Y\neq X}^{N}\sum_{i}^{N}[(d_{XY}^{new} - d_{XY}^{old})(<V_{X_i}^{new}$$
$$-V_{X_i}^{old}>)][(<V_{Y,i+1}+V_{Y,i-1}>)^{new}$$
$$-(<V_{Y,i+1}+V_{Y,i-1}>)^{old}] \qquad (2.10)$$

If the energy difference ΔE ≤ 0, the probability of accepting the solution becomes 1 otherwise it is $P_{mp} = e^{-\Delta E/T}$. On each iteration with the highest probability of accepting the solution, constraint parameter of Annealing Schedule is defined as,

$$T = d_{\max}^{new} \quad (2.11)$$

On each iteration the constraint parameter is being changed to the minimum value with respect to the previous value. Hence, each time the network determines the stable condition with the new value of energy function E and constraints parameter T and when $\Delta E \leq 0$. The neurons produce the stable states that represent the position of the city with minimum distance with respect to the previous position. So, at the stability with $\Delta E \leq 0$ the state of the neurons can be defined from the equation (2.8) as,

$$<Vx_i> = \tanh\left[-\frac{1}{d_{\max}^{new}} \frac{\partial[\frac{d_{\max}^{new}}{2}\sum_{i}^{N}\sum_{j\neq i}^{N}\sum_{X}^{N}<Vx_i><Vx_j> + \frac{1}{2}\sum_{X}^{N}\sum_{Y\neq X}^{N}\sum_{i}^{N} d_{XY} <V_{X_i}>(<V_{Y,i+1}+V_{Y,i-1}>)]}{\partial <V_{X_i}>}\right] \quad (2.12)$$

The Mean field for the neuron (X, i), Where output$<V_{Xi}>$ can be interpreted as the probability of finding city X in the i^{th} tour position as defined from the equation (2.3) as,

$$E_{X_i} = d_{\max}^{new}\sum_{Y\neq X}^{N}<V_{X_i}> + \sum_{Y\neq X}^{N} d_{XY}(<V_{Y,i+1}+V_{Y,i-1}>) \quad (2.13)$$

Thus, the entire process continues until fixed point is found for the every value of constraint parameter T. In this process of selecting a fixed point, change in the energy is computed and the probability of accepting the point in the minimum distance path can be found. The field for the selected points will also be computed. This entire process will continue for every schedule of the constraints parameter T, as it is changing with the value of $d_{\max}^{new}$, until the T reaches to the final value.

ALGORITHM

The algorithm of the entire process can be proposed as:

1. Initialize all neurons on the stable state as;

$$V_i(t+1) = f[\sum_{j\neq i} W_{ij}V(t)]$$

and initialize the weights and bias with the small random number. The value of T is initialized with any large random number.

2. Do until the fixed point is found.
   2.1 Randomly select a city say X.
   2.2 Compute the energy function as;

$$E_{TSP} = \frac{d\max}{2}\sum_{i}^{N}\sum_{j\neq i}^{N}\sum_{X}^{N}<V_{X_i}><V_{X_j}>$$
$$+\frac{1}{2}\sum_{X}^{N}\sum_{Y\neq X}^{N}\sum_{i}^{N} d_{XY} <V_{X_i}>(<V_{Y,i+1}+V_{Y,i-1}>)$$

Compute the state of the unit at equilibrium;

$$<Vx_i> = \tanh[-\frac{1}{d\max}\frac{\partial E_{TSP}}{\partial <V_{X_i}>}]$$

Also calculate the ΔE using equation (2.7)

2.3 if $\Delta E \le 0$ accept with P$_{<V_{X_i}>}^{(accept)}$ =1 and set T= d_{max} else P$_{<V_{X_i}>}^{(accept)}$ = exp (- ΔE /T)

Compute the Mean field as

$$Ex_i = d_{\max} \sum_{Y \neq X}^{N} < V_{X_i} > + d_{XY} (< V_{Y,i+1} + V_{Y,i-1} >)$$

(The average of the output values of accepted fixed point i.e. neurons)

3. If T reaches to the final value stop the processing otherwise decrease the T according to the annealing schedule and repeat the step 2.

3. CONCLUSIONS

In this paper, we presented the method of mean field approximation for determining the possible minimum length path for a traveling salesman. This minimum length path will satisfy all the constraints imposed on the route.

The following observation can be made from the solution:

1. The solution of the problem is obtained by determining the stable state i.e. the average function of the neurons of the network using stochastic asynchronous relation procedure with an annealing schedule.
2. It is also true that this neural network approach does not yield minimum cost function some as the time.
3. For the large number of cities, the optimum solution for this problem depends on the chain of the parameter used for the constraint terms and for implementing the annealing process.
4. The new energy function is being used that is different from the old energy function proposed by Hopfield and Tank. This new function of Mean field approximation involving the less term with respect to old energy function.
5. The constraint parameter T of the Annealing Schedule will be change on each iteration of the process with the d_{max} distance, which is slightly larger distance between cities that have selected for the path.

Thus energy iteration will be schedule with the optimize value of T.

Although the algorithm and the energy function can successfully apply to the TSP problem and almost optimal solution could be found with 30 cities problem. The more experiments and analytical investigation are still required for increase the efficiency and speed of the solution.

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