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ANALYSIS OF GOAL PROGRAMMING PROCEDURE FOR FUZZY MULTI-OBJECTIVE PROGRAMMING PROBLEM

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ABSTRACT: The fundamental concept in Goal programming (GP) is to incorporate all goals of the decision-maker in the model formulation. GP can handle single or multiple goals. In classical theory, a decision can be characterized by a set of decision alternatives; a set of state of nature a relation assigning to each pair of decision and state a result and finally the utility function which orders the result according to their desirability. When deciding under certainty, the decision makers know which state to expect and he chosen the decision alternatives with the highest priority. The decision making contains a number of objectives and a number of constraints. In 1972 Camroon B, consider this classical model of decision making in fuzzy environment. They consider the situation of decision making under certainty. In which the objective function as well as the constraint are fuzzy. A decision in the fuzzy environment is defined by analogy to non-fuzzy environments as the selection of activities that simultaneously satisfied objective function and constraint.

KEYWORDS: Goal programming, Fuzzy environment

INTRODUCTION

This approach was first introduced by Charnes and Copper and then developed by Ijiri and Inzio. The main idea behind the goal programming is to find a best possible satisfactory solution of multi objective optimization problem. In goal programming, various goals are expressed in different units of measurement such as Rupees, hours, tones etc. Many times, the multiple goals are conflicting each other and one can be achieved at the cost of other so we choose a compromise solution among these goals. e.g.: A politician promises to decrease the country's debts and also promises to give income tax relief. In GP, all the goal programming constraints should be linear form.

REVIEW OF LITEARTURE

In reality, the decision-maker generally chooses the achievement of certain goals at the expense of others. Therefore, GP requires an ordinal ranking of the goals in order of importance by the decision-maker. The solution process then satisfies goals beginning with the goal with the highest priority.

$$\min Z = \Sigma = \Sigma w_i \left(d_i^+ + d_i^-\right)$$

$$\text{s. t. } \Sigma a_{ij}.x_j + d_i^- - dd_i^+ = b_i \forall \perp$$

$$X_j, d_i^-, d_i^+ \geq 0 \ \forall \ i \ j$$

Where x_j represents a decision variable, w_i represents the weights attached to goal i, and d_i^- ,and d_i^+ represent the under achievement and over achievement of a goal i respectively. GP, however, due to the nature of its objective function sometimes tends to overachieve certain goals while underachieving others. Goal interval programming (Charnes and Cooper (1977)) addresses this shortcoming by specifying an interval within which all points are equally desirable towards achievement of the target goal.

Narsimhan (1980) was the first to integrate the concepts of fuzzy set theory and goal programming. Tiwari et al. (1986) provide an extensive review of the various facets of fuzzy goal programming that have been researched by Hannan (1981, 1982), Narsimhan (1981), Ignizio (1982), Tiwari et al. (1985, 1986), and Rubin and Narsimhan (1984). Multi-criteria decision problems generally involve the resolution of multiple conflicting goals to achieve a "satisfycing" solution rather than maximization objectives given a suitable aspiration level for each objective. The generalized goal programming approach seeks to minimize the negative and positive (over achievement) deviations from the goal targets. However, in most real life situations the aspiration levels for some or all objectives typically have an imprecise nature. For example, the profit of a company should be around 2 million dollars.

Bellman and Zadeh (1970) extended fuzzy set theory and developed a framework for decision-making in a fuzzy environment. The realm of fuzzy decision making is discussed in this section. The literature review is divided into the following subsections.

First, the mathematical concepts and notions of fuzzy set theory are introduced. Second, the linkage of fuzzy set theory and decision-making is established.

In this subsection the definitions of fuzzy goal, fuzzy constraint, fuzzy decision, and optimal fuzzy decision are outlined. Third, the connection between fuzzy decision making and linear programming is discussed.

In particular the model formulation proposed by Zimmermann (1976) and adapted to DEA by Sengupta (1992) is illustrated. Sengupta's (1992) formulation is adapted to the GoDEA model (Athanassopoulos (1995) and developed in this research to provide a fuzzy decision-making environment incorporating goal programming and data envelopment analysis.

GENERAL IDEA OF THE VERTICAL HANDOFF DECISION ALGORITHM (VHDA)

A vertical handoff decision in a next generation wireless network environment (including WWAN, WLAN, WiMAX and Digital Video Broadcasting) must solve the following problem: given a mobile user equipped with a contemporary multi-interfaced mobile device connected to an access network, determine whether a vertical handoff should be initiated and dynamically select the optimum network connection from the available access network technologies to continue with an existing service or begin another service. Hence, our proposed VHDA consists of two parts: (a) A Fuzzy Logic Handoff Initiation Algorithm which uses a fuzzy logic inference system (FIS) to process a multicriteria vertical handoff initiation metrics, and (b) An Access Network Selection Algorithm which applies a unique fuzzy multiple attribute decision making (FMADM) access network selection function to select a suitable wireless access network. The vertical handoff decision function is triggered when any of the following events occur: (a) when the availability of a new attachment point or the unavailability of an old one is detected, and (b) when the user changes his/her profile, and thus altering the weights associated with the network selection attributes. Then the two-part algorithm is executed for the purpose of finding the optimum access network for the possible handoff of the already running services to the optimum target network.

The access network selection scheme involves decision making – a process of choosing among alternative courses of action for the purpose of attaining a goal or goals – in a fuzzy environment. It can be solved using FMADM which deals with the problem of choosing an alternative from a set of alternatives based on the classification of their imprecise attributes. The multiple attribute defined access network selection function selects the best access network that is optimized to the user's location, device conditions, service and application requirements, cost of service and throughput. The block diagram shown in Figure 1 describes the vertical handoff decision algorithm.

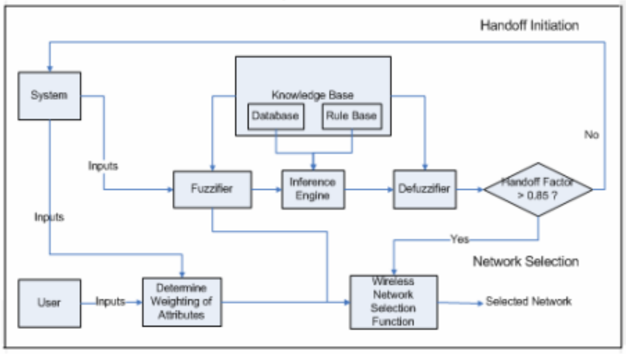

Fig.-1

HANDOFF INITIATION ALGORITHM

Vertical handoff is more complex because an MT can maintain connectivity to many overlaying networks that each offer varying QoS. Computing and choosing the correct time to initiate vertical handoff reduces subsequent handoffs, improves QoS, and limits the data signaling and rerouting that is inherent in the handoff process. To process vertical related parameters, we use fuzzy logic, which uses approximate modes of reasoning to tolerate vague and imprecise data. Fuzzy logic inference systems express mapping rules in terms of linguistic language.

CRISP SETS AND FUZZY SETS

A set which is a well defined collection of object is called a Crisp set, whereas a set which is not a well defined collection of object or which don't have the sharp boundary is called the fuzzy set.
There are two basic methods of writing a set:-

(i) Roster notation - $A = \{x_1, x_2, x_3\}$

(ii) Set builder notation - $B = \{x : x \varepsilon R(x)\}$

Every Crisp set can be written in form of fuzzy set by using a characteristic function A(x): $A \rightarrow \{0\ 1\}$ defined as -

$$X_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

∈ ∈

e.g.→ if $X = \{1,2,3,4,5,6,\ldots..9\}$

$A = \{1, 4, 7, 9\}$ then

$A = \{(1, 1), (2, 0), (3, 0), (4, 1), (5, 0), (6, 0), (7, 1), (8, 0), (9, 1)\}$

So characteristic function defined on a Crisp set can take only two values 0 & 1.

The first publication in the fuzzy set theory by Zadeh in 1965 and Goguen in 1967-1969. They generalized the Crisp function up to [0 1] which can be defined as -

$\mu_A : A \rightarrow [0\ 1]$ such that

$$\mu_A(x) = y \quad \forall \quad x \in A \quad \text{and} \quad 0 \leq y \leq 1$$

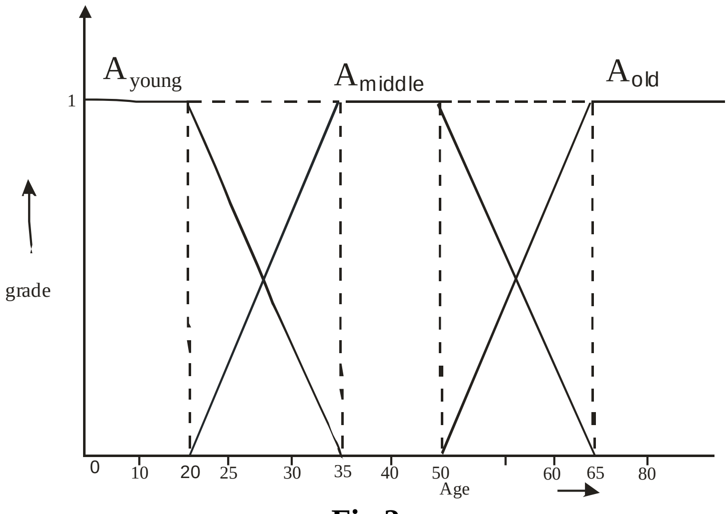

Fig 2

Here $\mu_A(x)$ is called the membership grade in which $x \in A$.

Example: Let us consider the three fuzzy sets for young age, middle age and old age denoted by A_Y, A_m, A_{old} respectively

$$\mu_{A_y}(x) = \begin{cases} 1 & if \quad x \le 20 \\ \dfrac{35-x}{15} & if \quad 20 \le x \le 35 \\ 0 & if \quad x \ge 35 \end{cases}$$

$$\mu_{A_m}(x) = \begin{cases} 0 & if \quad x \le 20 \\ \dfrac{x-20}{15} & if \quad 20 \le x \le 35 \\ 1 & if \quad 35 \le x \le 50 \\ \dfrac{65-x}{15} & if \quad 50 \le x \le 65 \\ 0 & if \quad x \ge 65 \end{cases}$$

$$\mu_{A_{old}}(x) = \begin{cases} 0 & if \quad x \le 50 \\ \dfrac{x-50}{15} & if \quad 50 \le x \le 65 \\ 1 & if \quad x \ge 65 \end{cases}$$

The short of membership drawing is known as trapezoidal membership function.

FUZZY PROGRAMMING

The FP approach for handling the multi objective problem was firstly introduced by Zimmermann, Narasimahn and Ignizio has investigated and developed the use of fuzzy set theory in solving problems with multiple goals. The approach of Zimmermann is the basis and the best of all these and other works and we will concentrate on presenting this approach.

Fuzzy linear programming using the min operator:-

Starting from the model B1 the adopted fuzzy version due to Zimmermann is-

$$CX \gtrsim Z$$
$$\text{s.t. } AX \lesssim \qquad [B7]$$

Where $\gtrsim$ nd$\lesssim$ are the fuzzification of $\ge$ and $\le$

Respectively. $\ge (\le)$ Means essentially greater (less) then

To solve B7, Zimmermann suggested using a linear membership function for each goal $\mu_{1k}(C_kX)$ where –

$$\mu_{1k}(c_kX) = \begin{cases} 1 & if \quad C_kX \ge \bar{Z}_k \\ 1 - \dfrac{\bar{Z}_k - C_kX}{d_{1k}} & \text{if} \quad Z_k - d_{1k}, \le C_kX \le \bar{Z}_k \\ 0 & if \quad C_kX \le \bar{Z}_k - d_{1k},\ k = 1, 2 \ldots.. k \end{cases}$$

And another linear membership function $\mu_{2i}(a_iX)$ for the i[th] constraint in the systum constraints $AX \le b$, where -

$$\mu_{2i}(a_ix) = \begin{cases} 1 & if \quad a_ix \le b_i \\ 1 - \dfrac{a_ix - b_i}{d_{2i}} & if \quad b_i \le a_ix \le b_i + d_{2i} \\ 0 & if \quad a_ix \ge b_i + d_{2i} \\ & \text{for } i = 1, 2 \ldots m \end{cases}$$

Where d_{1k} (k = 1, 2,--K) and d_{2i} (i = 1, 2--M) are subjectively chosen of admissible violations and a_i the i_{th} row of the matrix A.

Since $\mu_{1k}(C_k X)$ *and* $\mu_{2i}(aiX)$ express the satisfaction of the decision maker with the sol^n. they must be maximized

t.e. The problem is -

$$\max\{\mu_{11}(C_k X)--\mu_{1k}(C_k X),\mu_{21}(a_1 X),--\mu_{2m}(a_m X)\}$$

In one of the fuzzy set theorems, then the membership function of the intersection of any two sets is the minimum membership function of these sets. By using this theorem the problem is converted to-

$$\max_x[\min(\mu_{11}(C_1 X)--\mu_{1k}(C_k X),\mu_{2m}(a_1 X),--\mu_{2m}(a_m X)]$$

This fuzzy program can be written as -

Max Y, [B8]

s.t.

$$y \le 1-(\bar{Z}_k - C_k X)/_{d_{1k}}, k = 1,2,3-K.$$

$$y \le 1-(a_i x - b_i)/d_{2i}, i = 1,2--M.$$

$$y \ge 0, X \ge 0$$

The program B8 is a linear program that can be solved using simplex method.

FUZZY GOAL PROGRAMMING

The author developed a new approach for transforming B7 to a linear goal program. This approach depends on the fact that the maximum value of any membership function is 1. Hence maximizing any of them is equivalent to making them as close as possible to 1 by minimizing the its negative deviational variable from 1.

In this sense, the problem is converted to a G.P. that can take any of the forms B3, B4, and B5 B6.

By applying the form B3 to the fuzzy program B7 using the definition of $\mu_{1k}(C_k X)$ *and* $\mu_{2i}(a_i x)$ the following program can be obtained.

Min ϕ [B9]

s.t.

$$1-(\bar{Z}_k - C_k X)/d_{1k} + k + n_{1k} - p_{1k} = 1$$

$$1-(a_i x - bi)/d_{2i} + n_{2i} - p_{2i} = 1$$

$$\phi \ge n_{1k}$$

$$\phi \ge n_{2i}$$

$$X \ge 0,\ n_{1k} \ge 0,\ n_{2i} \ge 0,\ p_{1k} \ge 0, p_{2i} \ge 0$$

$$n_{1k} p_{1k} = 0,\ n_{2i}.p_{2i} = 0,\ k = 1,2,3-K.$$

B9 is a linear program that can be solved using the simplex method.

Now, we will use the goal programming technique discussed in model B4, B5, B6 to solve the linear program B9. Now, applying B4 to solve B9, We can get the following linear program.

Min [B10]

$$[\sum_{k=1}^{k} n_{1k} + \sum_{i=1}^{m} n_{2i}]$$
$$s.t.$$
$$1-(\overline{Z}_k - C_k X)/d_{1k}k - n_{1k} - p_{1k} = 1$$
$$1-(aiX - bi)/d_{2i} + n_{2i} - p_{2i} = 1$$
$$\phi \geq n_{1k}.\ \ \phi \geq n_{2i}$$
$$X \geq 0.\ \ n_{1k} \geq 0,\ \ p_{2k} \geq 0,\ \ n_{1k}.p_{1k} = 0, k = 1,2,3 - K$$
$$n_{2i} \geq 0,\ \ p_{2i} \geq 0,\ \ n_{2i}.p_{2i} = 0,\ \ i = 1,2,3 - M.$$

Now, applying B5 to solve B9, up can get the following linear program-

[B11]

$$\min \sum_{k=1}^{k} w_{1k} n_{1k} + \sum_{i=1}^{m} w_{2i} n_{2i}$$
$$s.t.$$
$$1-(\overline{Z}_k - C_k X)/d_{1k} + n_{1k} - p_{1k} = 1$$
$$1-(aiX - bi)/d_{2i} + n_{2i} - p_{2i} = 1$$
$$n_{1k} \geq 0,\ \ p_{1k} \geq 0,\ \ n_{1k}.p_{1k} = 0, k = 1,2,3 - K$$
$$p_{2i} \geq 0,\ \ n_{2i} \geq 0,\ \ n_{2i}.p_{2i} = 0,\ \ i = 1,2,3 - M.$$

Finally, if the decision maker can assign priority ranking for each goal, then using B6 to solve B9, we get the following linear program-

[B12]

$$\min\ \ a = \{\sum_{k=1}^{k} w_{1k} n_{1k} + \sum_{i=1}^{m} w_{2i} n_{2i}, i = 1,2 - I\}$$
$$s.t.$$
$$1-(\overline{Z}_k - C_k X)/d_{1k} + n_{1k} - p_{1k} = 1$$
$$1-(aiX - bi)/d_{2i} + n_{2i} - p_{2i} = 1$$
$$n_{1k} \geq 0,\ \ p_{1k} \geq 0,\ \ n_{1k}.p_{1k} = 0, k = 1,2,-K$$
$$n_{2i} \geq 0,\ \ p_{2i} \geq 0,\ \ n_{2i}.p_{2i} = 0,\ \ i = 1,2,-M.$$

The model B10, B11, B12 represents the different types of fuzzy program in the case having linear membership function and the fuzziness is in the right hand side of both the goals and constraints.

RELATIONSHIP BETWEEN GP AND FP

GP and FP are two approaches for solving the multi-objective optimization problem B1, both of them need an aspiration level for each objective. These expression levels are determined either by the decision maker or the decision analyst. In addition to the aspiration level of the goals, FP need admissible Violation constants d_{1k} for each goal. The larger d_{1k} indicates less important Kth goal. Accordingly, the following theorem can be stated about the relationship between FP and GP.

Result: Every fuzzy linear program has an equivalent weighted linear goal program where the weights are the reciprocals of the admissible violation constants.

Proof: Without loose of generality, we will assume that the fuzziness in the aspiration levels of the goals and not in the system constraints (the goals and constraints are treated in the some way in FP).

We will prove that B8 is equivalent to B3 with weighted deviational variables the weights are $1/d_{1K}$ (k = 1, 2, 3--K).

Starting with B5, it can be rewritten as-

Min [1-y]

s.t. $1-y \geq (\overline{Z}_k - C_k X)/d_{1k}$

$AX \leq b \;, X \geq 0$

Since y is a membership function. $y \leq 1$ Which means that $1-y \geq 0$. Let [1-y] = U then the problem can be transformed as

$$\begin{aligned} &\min u && \text{[B13]} \\ &s.t. \\ &u \geq (\overline{Z}_k - C_k X)/d_{1k} \\ &AX \leq b,\ \ X \geq 0,\ \ u \geq 0 \end{aligned}$$

From B13 $u \geq \max(0, \dfrac{\overline{Z}_k - C_k X}{d_{1k}})$ and by using the definition of negative deviational variables in (1) it can be obtained as

$$\begin{aligned} &u \geq n_{1k} where - \\ &\frac{C_k x}{d_{1k}} + n_{1k} - p_{1k} = \frac{\overline{Z}_k}{d_{1k}} \\ &\rightarrow C_k X + d_{1k} n_{1k} - d_{1k} p_{1k} = \overline{Z}_k \end{aligned}$$

The full program B13 can be rewritten as-

$$\begin{aligned} &\min u && \text{[B14]} \\ &st. \\ &AX \leq b \\ &C_k X + d_{1k} n_{1k} - d_{1k} p_{1k} = \overline{Z}_k \\ &X \geq 0,\ \ n_{1k} \geq 0,\ \ p_{1k} \geq 0\ \ ,\ \ n_{1k}.p_{1k} = 0 \\ &k = 1,2,3--K. \end{aligned}$$

B14 is equivalent to B3 with $n_k = d_{1k}.n_{1k} \rightarrow n_{1k} = \dfrac{n_k}{d_{1k}}$ thanB8 is equivalent to B3 with weighted deviational variables where the weights equal $1/d_{1k}$. The proof is completed by considering the relationship with GP in its min-max form and fuzzy linear program using the min operator.

CONCLUSION

The linear multi objective program can be solved using many approaches. In this paper GP and FP are consider as two important approaches for transforming the multiple objective model to a single objective program or an ordered sequential multi objective one. This paper made a survey for the formulation of GP and FP and developed some new FP forms by using the sense of GP. The relationship between all forms of FP and GP is investigated within this paper. It is proved that every FP is a GP with some weights assigned to the deviational variables in the objective function. These weights are reciprocal of the admissible violence constants. The FP considered in this paper has fuzziness in the aspiration levels. i.e. To get a sol_n that makes the sol_n as close as possible to a specific goal within a certain limit.

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