International Journal of Engineering Research & Management Technology
ISSN: 2348-4039
July-2015 Volume 2, Issue-4
Email: editor@ijermt.org www.ijermt.org

A Brief Study of Fuzzy Decision Making and Fuzzy Linear Programming

Kallu Ram Meena
Research Scholar
Sunrise University, Alwar, Rajasthan

Dr. Kapil Kumar Bansal
Supervisor
Sunrise University, Alwar, Rajasthan

ABSTRACT:
Mathematical programming models for agricultural planning problem have been widely used since heady demonstrated the use of linear programming (LP) for land allocation to crop planning problems. From 1960s to mid 1980s, LP models of different farm planning problems have been extensively studied. The potential use of LP for agricultural planning problem has been surveyed by glen in 1987. Since LP is a single objective optimization technique and most of the farm planning problems are multi-objective in nature.

KEY WORDS: linear programming

INTRODUCTION:
The goal programming approach, one of the prominent tools for multi-objective decision analysis, to land allocation planning problem for optimal production of several crops was first introduced by wheeler and Russel in 1977. The application potential of GP to farm planning problems has been surveyed by Romero. The use of preemptive priority based GP to land use planning problem have been discussed by Pal and Basu. Although GP has been widely used for farm planning problems, the main weakness of conventional GP formulation is that all the parameters of the problem need to be specified precisely in the planning environment. But in most of the practical decision problem, they are often imprecisely defined due to the expert's ambiguous understanding of the nature of them. So assigning of definite aspiration level to the goals of the problem frequently Creates decision variable in most of the farm planning situations. To overcome the above difficulty, the concept of fuzzy sets, initially proposed by Zadah, has been introduced to the field of multi-objective optimization problem. The use of fuzzy linear programming (FLP) to farm planning problem has been discussed by slowinski. The fuzzy goal programming approach (FGP) to Crop planning problems in the environment of Crisp resource constraints has been recently studied by Pal and Moitra. However in contrast to LP and GP approach, fuzzy programming (FP) approach to farm planning problems has not been appeared extensively in the literature.

CRISP SETS AND FUZZY SETS:-
A set which is a well defined collection of object is called a Crisp set, whereas a set which is not a well defined collection of object or which don't have the sharp boundary is called the fuzzy set.
There are two basic methods of writing a set:-

(i) Roster notation - $A = \{x_{\square}, x_{\square}, x_{\square}\}$

(ii) Set builder notation - $B = \{x : x \square\ R(x)\}$

Every Crisp set can be written in form of fuzzy set by using a characteristic function A(x): A →{0 1} defined as -

$X_A(x) =$

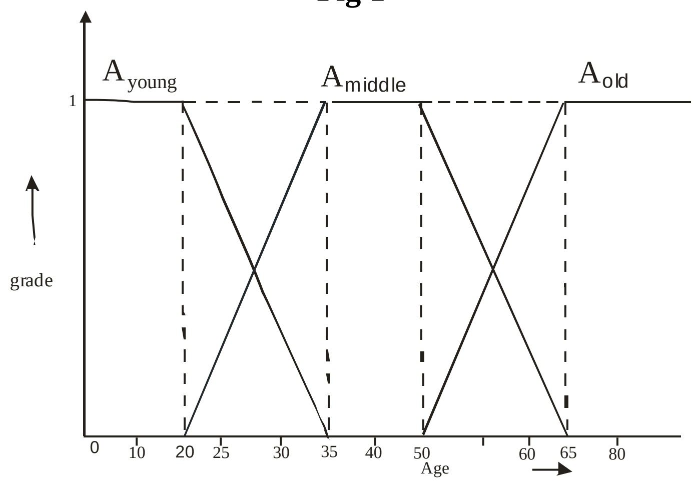
e.g.→ if X = { 1,2,3,4,5,6,…..9}
A = {1, 4, 7, 9} then
A = {(1, 1), (2, 0), (3, 0), (4, 1), (5, 0), (6, 0), (7, 1), (8, 0), (9, 1)}

So characteristic function defined on a Crisp set can take only two values 0 & 1.
The first publication in the fuzzy set theory by Zadeh in 1965 and Goguen in 1967-1969 They generalized the Crisp function upto [0 1] which can be defined as -

μ□ □ □ □ A→ [0 1] such that

μ□ □ x□ □ □ □□y x□ □ □ □ A and 0 ≤ y ≤ 1

Fig 1



Here μA□ x□ □ is called the membership grade in which x □ □ A. ∈
Example: Let us consider the three fuzzy sets for young age, middle age and old age denoted by A_Y, A_m, A_{old} respectively

$$\mu_{A_y}(x) = \begin{cases} 1 & if \quad x \le 20 \\ \dfrac{35-x}{15} & if \quad 20 \le x \le 35 \\ 0 & if \quad x \ge 35 \end{cases}$$

$$\mu_{A_m}(x) = \begin{cases} 0 & if \quad x \le 20 \\ \dfrac{x-20}{15} & if \quad 20 \le x \le 35 \\ 1 & if \quad 35 \le x \le 50 \\ \dfrac{65-x}{15} & if \quad 50 \le x \le 65 \\ 0 & if \quad x \ge 65 \end{cases}$$

$$\mu_{A_{old}}(x) = \begin{cases} 0 & if \quad x \le 50 \\ \dfrac{x-50}{15} & if \quad 50 \le x \le 65 \\ 1 & if \quad x \ge 65 \end{cases}$$

The short of membership drawing is known as trapezoidal membership function.

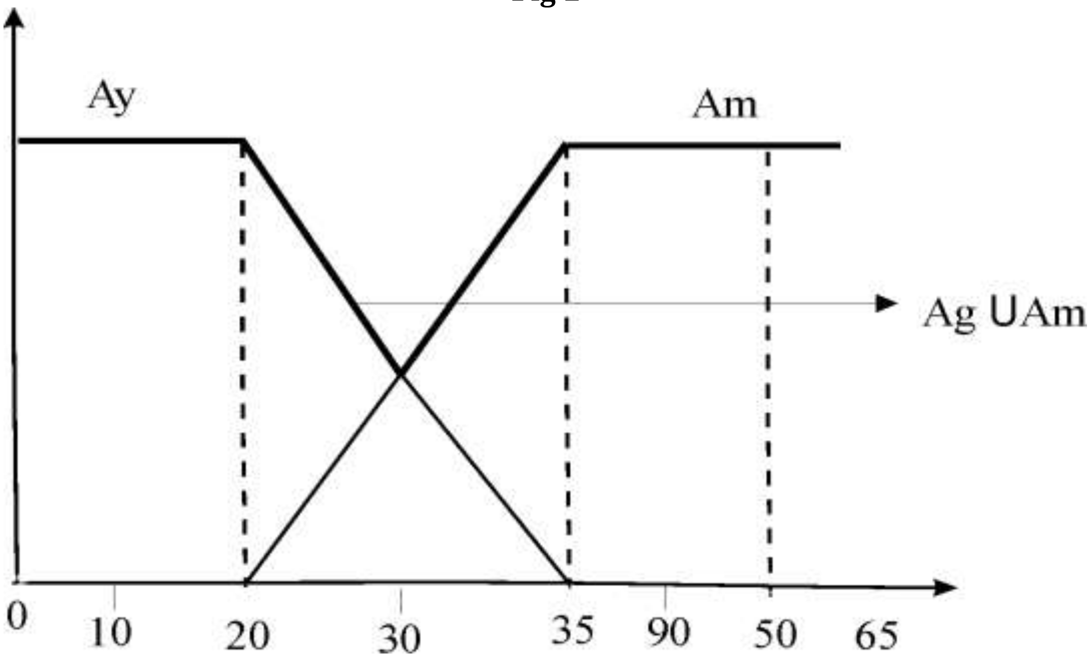
SOME BASIC DEFINITIONS:
STANDARD FUZZY UNION:
The union of two fuzzy set is again a fuzzy set which can be defined as follow-

Where $$A \cup B = \{(x, \mu_{A \cup B}(x)): x \in X\}$$

e.g.: if A_y and A_m denote the fuzzy set for young age and middle age respectively then there union is expressed by darken lines.

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

Fig 2



STANDARD FUZZY INTERSECTION:

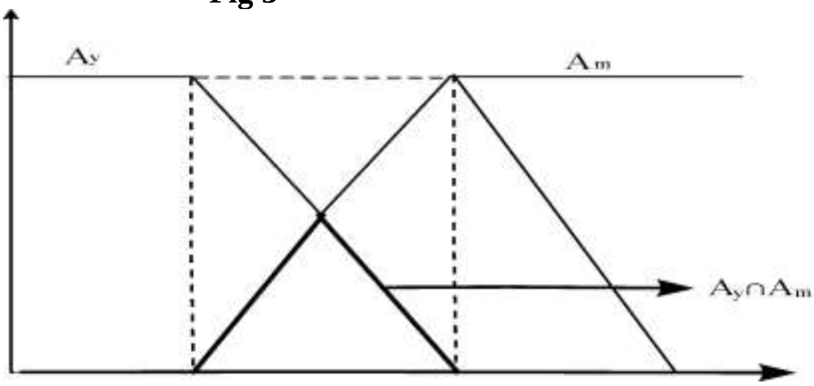
The intersection of two fuzzy set is an another fuzzy set which can be defined as follow –

$$A \cap B = \{(x.\ \mu_{A \cap B}(x)) : x \in X\}$$

Where

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$

Fig 3



Standard fuzzy complement:

if A be a fuzzy set defined as –

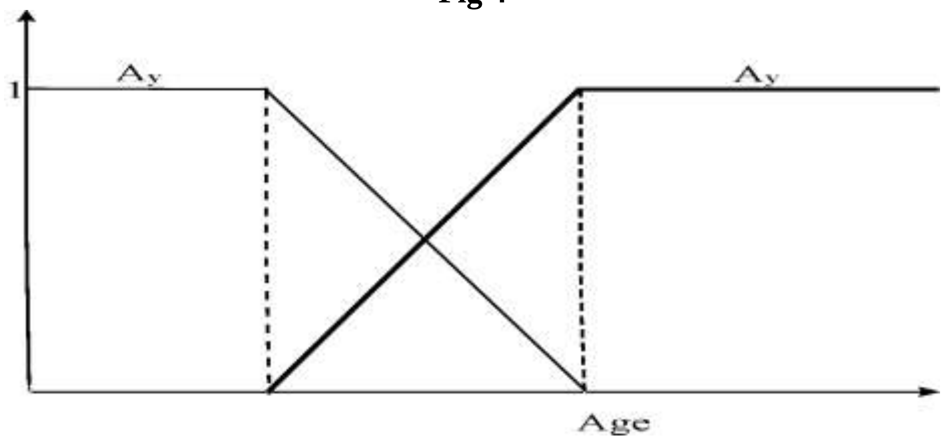
$$A = \{(x, \mu_A(x)) : x \in X\}$$

Then its standard fuzzy complement is a fuzzy set s.t.

$\overline{A} = \{(x, \mu_{\overline{A}}(x)) : x \in x\}$ Where $\mu_{\overline{A}}(x) = 1 - \mu_A(x)$

e.g.: Let A_y denoted the fuzzy set for young age then A_Y is denoted by darken lines

Fig 4



α - Cut of fuzzy set:

An α - Cut of a fuzzy set $\tilde{A}$ is denoted $^{\alpha}\tilde{A}$ is a Crisp set defined as –

$$^{\alpha}\tilde{A} = \{x \in X : \mu_{A}^{(x)} \geq \alpha\} \text{ Where and } \alpha \in [01]$$

Strong α - Cut of a fuzzy set:

A strong α - Cut of a fuzzy set, denoted by $^{\alpha+}A$ is defined as -

$$^{\alpha+}A = \{x \in X : \mu_{A}^{(x)} > \alpha\}$$

$$\text{Where } x \in X, \forall\ \alpha \in [0\ 1]$$

Support and Core of a fuzzy set

Let $\tilde{A}$ be a fuzzy set. Then support of $\tilde{A}$ is denoted by supp $\tilde{A}$ and defined as-

$$\text{supp}(\tilde{A}) = \{x \in X : \mu_A(x) > 0\}$$

Similarly

Core of $\tilde{A}$ is denoted by core of $\tilde{A}$ and is defined as-

$$Core(\tilde{A}) = \{x \in X : \mu_{\tilde{A}}(x) = 1\}$$

EXAMPLE:

$$\text{Let } \tilde{A} = \{\frac{x_1}{.1} + \frac{x_2}{0} + \frac{x_3}{.5} + \frac{x_4}{1} + \frac{x_5}{.7} + \frac{x_6}{1}\}$$

Then

$$Supp(\tilde{A}) = \{x_1, x_3, x_4, x_5, x_6\}$$

$$\text{Core}(\tilde{A}) = \{x_4, x_6\}$$

DECISION MAKING IN FUZZY ENVIRONMENT AND FUZZY LINEAR PROGRAMMING:
DECISION MAKING AND FUZZY DECISION:

In classical theory, a decision can be characterized by a set of decision alternatives; a set of state of nature a relation assigning to each pair of decision and state a result and finally the utility function which orders the result according to their desirability.

When deciding under certainty, the decision makers knows which state to expect and he chosen the decision alternatives with the highest priority.

The decision making contains a number of objectives and a number of constraints.

In 1970 Bell man and Zadeh consider this classical model of decision making in fuzzy environment. They consider the situation of decision making under certainty. In which the objective function as well as the constraint are fuzzy.

The objective function is characterized by its membership value and so by constraints.

A decision in the fuzzy environment is defined by analogy to non-fuzzy environments as the selection of activities that simultaneously satisfied objective function and constraint.

Assume that we are given a fuzzy goal $\tilde{G}$ and a fuzzy constraint in the $\tilde{C}$ space of alternatives X. then $\tilde{G}$ and $\tilde{C}$ combined form a decision $\tilde{D}$ which is a fuzzy set resulting from the intersection of and of $\tilde{G}$ and $\tilde{C}$ symbolically –

$$\tilde{D} = \tilde{G} \cap \tilde{C} \ and \ \mu_{\tilde{D}} = \min\{\mu_{\tilde{G}}, \mu_{\tilde{C}}\}$$

More generally, suppose we have n goals $\tilde{G}_1, \tilde{G}_2, \_ \tilde{G}_n$ and in constraints $\tilde{C}_1, \tilde{C}_2, \_ \tilde{C}_n$ then resultant decision is the intersection of the given goals G_1, G_2, _ _ G_n and given constraint C_1,C_2_ _C_n.

$$t.e. \quad \tilde{D} = \tilde{G}_1 \cap \tilde{G}_2 \cap - \cap \tilde{G}_n \cap \tilde{C}_2 \cap - \cap \tilde{C}_n$$

and correspondingly –

$$\mu_{\tilde{D}} = \min\{\mu_{\tilde{G}_1}, \mu_{\tilde{G}_2} - - \mu_{\tilde{G}_n}, \mu_{\tilde{C}_1} - - \mu_{\tilde{C}_n}\}$$

$$= \min\{\mu_{\tilde{G}i}, \mu_{Ci} : 1 \le i \le n\}$$

LINEAR PROGRAMMING AND FUZZY LINEAR PROGRAMMING:

The general L.P.P. can be defined as

Optimize Z = CX

Such that

AX $(\le = \ge) b$

and $X \ge 0$

It is a special type of decision making problem. The goal is defined by the objective function and the type of decision is the decision making under these conditions –

The classical model of linear programming models can be stated as-

Max Z = $C^T X$

Subject to –

$$AX \le b, \quad X \ge 0$$

$$Where \ C, X \in R^n, b \in R^m$$

In classical sense, the coefficient of A. b and c are crisp numbers and $\le$ is meant in a Crisp sense.

GOAL PROGRAMMING:

This approach was first introduced by Charnes and Copper and then developed by Ijiri and Inzio.

The main idea behind the goal programming is to find a best possible satisfactory solution of multi objective optimization problem.

In goal programming, various goals are expressed in different units of measurement such as Rupees, hours, tones etc.

Many times, the multiple goals are conflicting each other and one can be achieved at the cost of other so we choose a compromise solution among these goals.

e.g.: A politician promises to decrease the country's debt's and also promises to give income tax relief. In GP, all the goal programming constraints should be linear form.

Example:

TAX PLANNING PROBLEM:

Fairville is a small city with a population of about 20000 Residents. The city counsel is in the process of developing an equitable city tax rate table. The annual taxation based for real estate property is Rs. 550 millions. The annual taxation based for food and drugs and for general sales is Rs. 35 millions and 55 millions respectively. Annual local gasoline consumption is estimated at 7.5 millions gallons.

Let and x_p, x_s, x_f denote the tax rate on estate property, food and drugs and for general sales. Let x_g denote the general sales for gasoline oil per millions then four goals can be expressed as –

$$550x_p + 35x_f + 55x_s + .075x_g \geq 16 \quad (1)$$

$$35x_f \leq .1(550x_p + 35x_f + 55x_s + .075x_g) \quad (2)$$

$$55x_s \leq .2(550x_s + 35x_f + 55x_s + .075x_g) \quad (3)$$

$$x_g \leq 2. \quad (4)$$

SOLUTION PROCESS:

There are two main approaches to solve a goal programming problem.

1.The weighted method.

2.The primitive method.

Both of these methods are based on converting the multiple objective one into single objective one.

(i) THE WEIGHTED METHOD:

A single objective function is formed as the weighed sum of the functions representing the goal.

Suppose the goal programming models has n goals and the ith goal is given by minimize Gi : I = 1, 2 --n.

The combined objective function used in the weighted method is then defined as -

Minimize Z = $W_1G_1+W_2G_2$ + -- W_nG_n

The parameters w_i : i = 1, 2 -n are positive weights that reflects the decision makers performance regarding the relative important of each goals.

If w_i = 1 for each i signifies that all goals carry equal weights. The determination of specific values of these goals weights is subjective. There are serval methods to choose weights.

(ii) THE PRIMITIVE METHOD:

In primitive method the decision makers must rank the goals of the problem in order of Importance.

Given n goal situations, the objective of the problem are written as-

Minimize $G_1 = P_1$ (Higher priority)

Minimize $G_n = P_n$ (Lowest priority)

The variable P_i is the component of the deviational variables s_i^- and s_i^+ that represents the goal i.

The solution procedures consider one goal at a time, starting with a higher priority goal G, and terminating with lower priority Goal G_n. The process is carried out such that the solution obtained from a lower priority goal never degenerate the higher priority goal solution.

REFERENCES:

1. Charnes A. and Copper W.W. "Management models and the industrial applications of linear programming" Wiley, New York 1968.
2. District statistical hand book, Nadia (1999-2000), Department of Bereau of applied Economics and statistics, Govt. of WB, India.
3. Goicoechea A, Hansen D.R. and Duckstein, "Multiple objective decision analysis with engineering and business applications" Wiley, New York 1982.
4. Pal B.B. Basu I. "Selection of appreciate priority structure for optimal land allocation in agricultural planning threw goal program" Indian Journal of Agricultural Economics 1996, 51: 342-3540.
5. Romero C. "A survey of generalized goal programming." European Journal of Operation Research, 1986; 25: 183-191.
6. Bansal,K.K. & Sharma, M.K.(2015): Fuzzy Analysis of Shortest Job First, International Journal of Engineering Research & Management Technology
7. Wheeler B.M. Russel J.R.M, ''Goal programming an agricultured planning operation research'' quarterly 1977; 28: 21-32.
8. Zimmermann H-J. "Fuzzy programming and linear programming with serval objective functions" Fuzzy sets and system 1978; 1: 45-55.
9. W. F. Abd El-Wahed, Sang M. Lee, Interactive fuzzy goal programming for multi-objective transportation problems, The nternational Journal of Management Science (2006), 158-166.
10. R.N. Tiwari, S. Dharma, J.R. Rao, Priority structure in fuzzy goal programming, Fuzzy Sets and Systems (1985), 251-259.
11. C. Romero, Handbook of Critical Issues in Goal Programming, Oxford, New York, Pergamon Press (1991).
    Charnes, W.W. Cooper, Goal programming and multipleobjective optimisations (Part I), European Journal of Operational Research, 1 (1977), 39-54