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Inventory Model for Deteriorating Items with Non-Linear Production Rate Depending On Linear Demand Rate

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ABSTRACT:
In this paper, the inventory model for deteriorating items with non-linear production rate depending on linear demand rate has been discussed. Cost minimization technique has been used to obtain the optimal values of parameters. The total expected cost, maximum inventory level and unfilled backlog order have been obtained as the important operating characteristic of the model.

Key-words: Demand rate, production rate and deterioration.

1 INTRODUCTION:

Several mathematical models for controlling inventory have been developed by researchers. Few works has been done on developing the production inventory policy for deteriorating products. In practice, the demand may influence production. If the demand decreases, the manufacturer may decrease the production to avoid unnecessary inventory.

Ghare & Schrader (1963) developed an economic order quantity (EOQ) model by a negative exponential deterioration. They assumed that the instantaneous distribution is constant. **Mandal & Phaujdar** (1989), **Goswami & Chaudhuri** (1991, 92) and **Bose, et al** (1995) assumed either instantaneous or finite production with different assumptions on deterioration pattern. These models were developed for the simple cases of constant deterioration rate and time proportional deterioration rate. In standard EOQ models, a constant and known demand rate is assumed over an infinite time horizon. In practice most of the items experience a stable demand only during the saturation phase of their life cycle and for a definite period of time. Some researchers extended the EOQ model to accommodate time varying demand pattern. **Goswami & Chaudhuri** (1991, 92), **Bhunia & Maiti** (1997) and **Bose, et al** (1995) assumed a linear trend of demand. **Mandal et al** (1998), **Kar et al** (2001) and **Santosh Kumar et al** (2001) discussed the inventory models of deteriorating items with different demand rate.

Goswami & Chaudhuri (1992) developed an order level inventory model for deteriorating items in which the finite production rate is proportional to the time dependent demand rate. **Bhunia & Maiti** (1997) presented inventory models in which the production rate depends on either the on hand inventory or the demand. In the above mentioned models, the deterministic inventory models with linear production rate and varying demand rate including exponential declining demand have been attempted to obtain total average cost and other characteristic of the models. **Mishra, Himanshu Pandey and Balram** (2006) presented a probabilistic inventory model with non-linear production rate and uniformly distribution and declining demand rates. They obtained total expected cost, maximum inventory level and unfilled backlog orders and other characteristic of the model. They considered the demand rate $D(t)=\frac{1}{\beta-\alpha}$, $\alpha<t<\beta$ and 0 otherwise and the production rate

$P(t) = a + bD(t) + c[D(t)]^2$, a>0, $0 \le b \le 1$, $0 \le c \le 1$ and $P(t) > D(t)$. In this paper, the demand rate is taken constant and consequently the production rate is also constant. Thus there is nothing new except that demand exists during the period (α, β). Besides this there is no probabilistic concept equipped with any parameter of the inventory model. Thus the title of the research paper is not justified.

Sanjay Jain & Mukesh Kumar (2007) discussed an inventory model with inventory level dependent demand rate, shortages and decrease in demand. **Kharna et al.** (2010) developed an EOQ model with price and stock demand rate. **Hari Kishan, Megha Rani and Deep Shikha (2012)** discussed an inventory model of deteriorating products with life time under declining demand and permissible delay in payment.

Kundu et al. (2013) developed an EOQ model for time dependent deteriorating items with alternating demand rates allowing shortages and time discount.

In this paper, the inventory model for deteriorating items with non-linear production rate depending on linear demand rate has been discussed. The total expected cost, maximum inventory level and unfilled backlog order have been obtained as the important operating characteristic of the model.

2 ASSUMPTIONS AND NOTATIONS:

The mathematical model of the production inventory problem under consideration has been developed on the basis of the following assumptions:

(i) Demand rate D(t) at time t, $t \ge 0$ is given by $D(t) = \alpha t$, $\alpha > 0$.

(ii) Production rate P(t) at any instant t depends on demand rate and is given by $P(t) = a + bD(t) + c[D(t)]^2$, a>0, $b \ge 1$, $0 \le c \le 1$.

(iii) Deterioration of items is considered from that instant they have been received into the inventory.

(iv) Single item is considered over a period of T units subjected to a constant rate θ.

(v) Shortages are allowed and backlogged.

(vi) No replacement or repair of deteriorated items is made during a cycle.

The following notations are used in this paper:

(i) q(t): Inventory level at any time t, $t \ge 0$.

(ii) C: Set up cost for each new cycle.

(iii) C_i: Inventory carrying cost per unit per month.

(iv) C_s: Shortage cost per unit.

(v) C_d: The cost of deteriorated item.

(vi) T: Cycle time $(= T_1 + T_2 + T_3 + T_4)$.

(vii) θ: The deterioration rate.

(viii) K: The total average cost of the system.

3 MATHEMATICAL MODELS AND ANALYSIS:

Let the initial stock be zero. The production inventory level starts at time t=0 and reaches to the level I_m, the maximum level after T_1 units of time. Then the production is stopped, the stock level declines continuously and becomes zero at time T_2. At this instant the shortage starts and accumulates to the level I_b at time T_3. At this instant, the fresh production starts to clear the backlog by the time t_4. Our objective is to find out the optimal values of T_1, T_2, T_3, T_4, q_m and q_b which minimize the total average cost K over the time horizon [0,T]. The model has been represented by figure 1 given below:

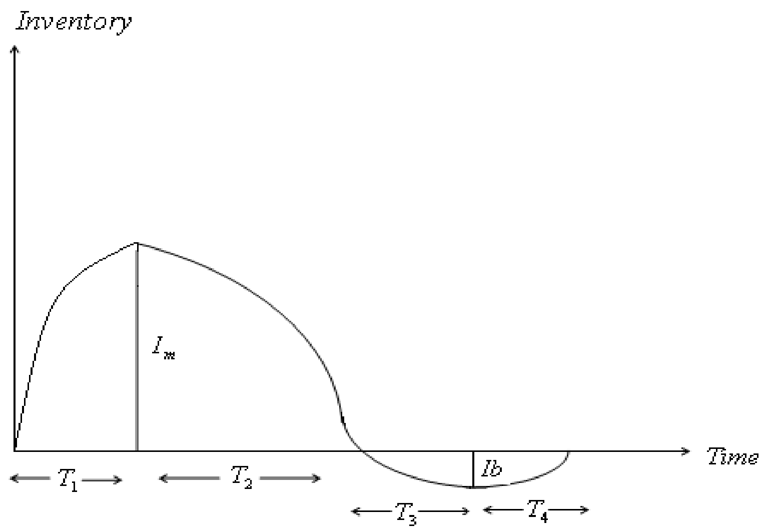

Figure 1

The governing differential equations of the stock status during the period $0 \le t \le T$ are given by

$$\frac{dq}{dt} = a + (b-1)\alpha t + c\alpha^2 t^2 - \theta q, \qquad 0 \le t \le T_1 \qquad \ldots(1)$$

$$\frac{dq}{dt} = -\alpha t - \theta q, \qquad 0 \le t \le T_2 \qquad \ldots(2)$$

$$\frac{dq}{dt} = -\alpha t, \qquad 0 \le t \le T_3 \qquad \ldots(3)$$

$$\frac{dq}{dt} = a + (b-1)\alpha t + c\alpha^2 t^2 - \theta q, \qquad 0 \le t \le T_4. \qquad \ldots(4)$$

The boundary conditions are

$$q(t) = 0 \text{ at } t = 0, T_1 + T_2 \text{ and T} \qquad \ldots(5)$$

$$q(t_1) = I_m \text{ and } q(T_1 + T_2 + T_3) = -I_b. \qquad \ldots(6)$$

From equation (1) we have

$$q(t)e^{\theta t} = \frac{\left[a + (b-1)\alpha t + c\alpha^2 t^2\right]}{\theta} e^{\theta t} - \frac{\left[(b-1)\alpha + 2c\alpha^2 t\right]}{\theta^2} e^{\theta t} + \frac{2c\alpha^2}{\theta^3} e^{\theta t} + c_1. \qquad \ldots(7)$$

Using the boundary condition (5) in the expression (7), we get

$$c_1 = -\frac{a}{\theta} + \frac{(b-1)\alpha}{\theta^2} - \frac{2c\alpha^2}{\theta^3}.$$

Substituting this value of c_1 in the expression (7), we get

$$q(t) = \frac{\left[a\left(1-e^{-\theta t}\right) + (b-1)\alpha t + c\alpha^2 t^2\right]}{\theta} - \frac{\left[(b-1)\alpha\left(1-e^{-\theta t}\right) + 2c\alpha^2 t\right]}{\theta^2} + \frac{2c\alpha^2\left(1-e^{-\theta t}\right)}{\theta^3}, \; 0 \le t \le T_1. \qquad \ldots(8)$$

Similarly the solutions of equations (2), (3) and (4) with the help of boundary conditions are given by

$$q(t)=\frac{\alpha}{\theta}\left(t_2 e^{\theta(T_2-t)}-t\right)+\frac{\alpha}{\theta^2}\left(1-e^{\theta(T_2-t)}\right), \qquad 0\le t\le T_2, \qquad \ldots(9)$$

$$q(t)=-\frac{\alpha t^2}{2}, \qquad 0\le t\le T_3 \qquad \ldots(10)$$

$$q(t)=\left[a+\frac{(b-1)}{2}(T_4+t)+\frac{c\alpha^2}{3}\left(T_4^2-T_4t+t^2\right)(T_4-t)\right]. \quad 0\le t\le T_4 \qquad \ldots(11)$$

Using the boundary condition (6) in the expressions (8) and (9), we get

$$q_m=\frac{\left[a\left(1-e^{-\theta T_1}\right)+(b-1)\alpha T_1+c\alpha^2T_1^2\right]}{\theta}-\frac{\left[(b-1)\alpha\left(1-e^{-\theta T_1}\right)+2c\alpha^2T_1\right]}{\theta^2}+\frac{2c\alpha^2\left(1-e^{-\theta T_1}\right)}{\theta^3}$$
$$=\frac{\alpha}{\theta}\left(t_2e^{\theta T_2}\right)+\frac{\alpha}{\theta^2}\left(1-e^{\theta T_2}\right). \qquad \ldots(12)$$

Using the boundary condition (6) in the expressions (10) and (11), we get

$$q_b=\frac{\alpha T_3^2}{2}=\left[a+\frac{(b-1)}{2}T_4+\frac{c\alpha^2}{3}T_4^3\right]T_4. \qquad \ldots(13)$$

From equation (12), we have

$$T_2\approx\sqrt{\frac{2}{\alpha}}[\{-a\left(1-e^{-\theta T_1}\right)-(b-1)\alpha T_1-2c\alpha^2T_1^2\}+\frac{(b-1)\alpha\left(1-e^{-\theta T_1}\right)+2c\alpha T_1^2}{\theta}-\frac{2c\alpha^2}{\theta^2}\left(1-e^{-\theta T_1}\right)]^{\frac{1}{2}}. \qquad \ldots(14)$$

From equation (13), we have

$$T_3=\sqrt{\frac{2}{\alpha}\left[a+\frac{(b-1)}{2}T_4+\frac{c\alpha^2}{3}T_4^2\right]T_4}. \qquad \ldots(15)$$

The inventory carrying cost is given by

$$C_i\left[\int_0^{T_1}q(t)dt+\int_0^{T_2}q(t)dt\right]$$
$$=\frac{C_1}{\theta}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}\left(t_1+\frac{e^{-\theta T_1}}{\theta}-\frac{1}{\theta}\right)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2\frac{T_1^2}{2}\right.$$
$$+\frac{c\alpha^2}{3\theta}T_1^3+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(e^{\theta T_2}-1\right)+T_2-\frac{\theta T_2^2}{2}]. \qquad \ldots(16)$$

The deterioration cost is given by

$$C_d\left[\int_0^{T_1}\theta q(t)dt+\int_0^{T_2}\theta q(t)dt\right]$$

$$= C_d\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}\left(T_1+\frac{e^{-\theta T_1}}{\theta}-\frac{1}{\theta}\right)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2\frac{T_1^2}{2}\right.$$
$$\left.+\frac{c\alpha^2}{3\theta}T_1^3+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(e^{\theta T_2}-1\right)+T_2-\frac{\theta T_2^2}{2}\right]. \qquad \ldots(17)$$

The shortage cost is given by

$$C_s\left[\int_0^{T_3}q(t)dt+\int_0^{T_4}q(t)dt\right]$$
$$= C_s\int_0^{T_3}\frac{\alpha t^2}{2}dt+\int_0^{T_4}\left[a+\frac{(b-1)}{2}(T_4+t)+\frac{c\alpha^2}{3}\left(T_4^2-T_4t+t^2\right)\right]dt$$
$$= C_s\left[\frac{\alpha T_3^3}{6}+aT_4+\frac{3}{4}(b-1)T_4^2+\frac{5c\alpha^2}{18}T_4^3\right]. \qquad \ldots(18)$$

Therefore the total expected cost of the inventory is given by

K=Set up cost +Inventory carrying cost +Deteriorating cost +Shortage cost.

$$= \frac{C}{T}+\frac{C_1}{\theta T}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}\left(T_1+\frac{e^{-\theta T_1}}{\theta}-\frac{1}{\theta}\right)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2\frac{T_1^2}{2}\right.$$
$$\left.+\frac{c\alpha^2}{3\theta}T_1^3+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(e^{\theta T_2}-1\right)+T_2-\frac{\theta T_2^2}{2}\right]$$
$$+\frac{C_d}{T}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}\left(T_1+\frac{e^{-\theta T_1}}{\theta}-\frac{1}{\theta}\right)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2\frac{T_1^2}{2}\right.$$
$$\left.+\frac{c\alpha^2}{3\theta}T_1^3+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(e^{\theta T_2}-1\right)+T_2-\frac{\theta T_2^2}{2}\right]$$
$$+C_s\left[\frac{\alpha T_3^3}{6}+aT_4+\frac{3}{4}(b-1)T_4^2+\frac{5c\alpha^2}{18}T_4^3\right]. \qquad \ldots(19)$$

It is extremely difficult to find the optimal solution of total expected cost K given by the equation (19). For extremely small values of θ one may use the approximate values of $e^{-\theta T_1}$ and $e^{-\theta T_2}$ given by

$$e^{-\theta T_1}\approx 1-\theta T_1+\frac{\theta^2T_1^2}{2}, \qquad \ldots(20)$$
$$e^{-\theta T_2}\approx 1-\theta T_2+\frac{\theta^2T_2^2}{2}. \qquad \ldots(21)$$

Using this approximate value in equation (19), we get

$K =$

$$\frac{C}{T}+\frac{C_1}{\theta T}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}\left(T_1+\frac{1-\theta T_1+\frac{\theta^2T_1^2}{2}}{\theta}-\frac{1}{\theta}\right)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2\frac{T_1^2}{2}\right.$$

$$+\frac{c\alpha^2}{3\theta}T_1^3+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(e^{\theta T_2}-1\right)+T_2-\frac{\theta T_2^2}{2}]+$$

$$\frac{C_d}{T}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}\left(t_1+\frac{1-\theta T_1+\frac{\theta^2T_1^2}{2}}{\theta}-\frac{1}{\theta}\right)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2\frac{T_1^2}{2}\right.$$

$$+\frac{c\alpha^2}{3\theta}T_1^3+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(\theta T_2+\frac{\theta^2T_2^2}{2}\right)-T_2+\frac{\theta T_2^2}{2}]$$

$$+C_s\left[\frac{\alpha T_3^3}{6}+aT_4+\frac{3}{4}(b-1)T_4^2+\frac{5c\alpha^2}{18}T_4^3\right]. \qquad \ldots(22)$$

Obviously, the cost function given by the equation (22) seems to be the function of four variables T_1, T_2, T_3 and T_4. But these four variables are not independent. They are related by the equations (14) and 15). Thus the cost function K may be considered as functions of two variables, say, T_1 and T_4. Therefore the optimal values of T_1 and T_4 are the solutions of the equations

$$\frac{\partial K}{\partial T_1}=0 \text{ and } \frac{\partial K}{\partial T_4}=0 \qquad \ldots(23)$$

Provided these values of T_1 and T_4 satisfy the condition

$$\frac{\partial^2 K}{\partial T_1^2}>0,\ \frac{\partial^2 K}{\partial T_4^2}>0 \text{ and } \frac{\partial^2 K}{\partial T_1^2}\frac{\partial^2 K}{\partial T_4^2}-\left(\frac{\partial^2 K}{\partial T_1\partial T_4}\right)^2>0. \qquad \ldots(24)$$

Differentiating equation (22) w.r.t. T_1 and T_4, we get

$$\frac{C_1}{\theta T}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}(\theta T_1)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2T_1\right.$$

$$+\frac{c\alpha^2}{\theta}T_1^2+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(e^{\theta T_2}-1\right)+T_2-\frac{\theta T_2^2}{2}]+\frac{C_d}{T}\left[\left\{a-\frac{(b-1)\alpha}{\theta}+\frac{2c\alpha^2}{\theta^2}\right\}(\theta T_1)+\left\{(b-1)\alpha-\frac{2c\alpha}{\theta}\right\}^2T_1\right.$$

$$+\frac{c\alpha^2}{\theta}T_1^2+\frac{\alpha}{\theta}\left(T_2-\frac{1}{\theta}\right)\left(\theta T_2+\frac{\theta^2T_2^2}{2}\right)-T_2+\frac{\theta T_2^2}{2}]=0. \qquad \ldots(25)$$

$$\left[\frac{\alpha T_3^2}{2}+a+\frac{3}{2}(b-1)T_4+\frac{5c\alpha^2}{6}T_4^2\right]=0. \qquad \ldots(26)$$

Thus we have two simultaneous non-linear equations given by (25) and (26). These equations can be solved by Newton-Rap son method to give the optimal values of T_1 and T_4. Consequently, we can obtain the optimal values of T_1, T_2, T_3, T_4 and K with the help of equations (14), (15) and (22) respectively.

4 CONCLUSIONS:

In this paper, the inventory model for deteriorating items with non-linear production rate depending on linear demand rate has been discussed. The total expected cost, maximum inventory level and unfilled backlog order have been obtained as the important operating characteristic of the model. This model can further be extended for other forms of the demand rate, the production rate and for variable deterioration. The model can also be generalized for multi items inventory system.

REFERENCES:

1. **Bhunia, A.K. and Maiti, M. (1998):** A two-warehouse inventory model for deteriorating items with a linear trend in demand and shortages. JORS, 49 (3), 287–292.
2. **Bose, S., Goswami, A., Chaudhuri, K.S. (1995):** An EOQ model for deteriorating items with linear time dependent demand rate and shortage under inflation and time discounting", JORS, 46, 777-782.
3. **Ghare, P.M. & Scharader, G.F. (1983):** A Model for Exponentially Decaying Inventories. Jour. Ind. Eng., 14, 238-243.
4. **Goswami, A. & Chaudhari, K. (1991):** EOQ Model for an Inventory with a Linear Trend in Demand and Finite Rate of Replenishment Considering Shortages. Int. J. Syst. Sci., 22,181-187.
5. **Goswami, A. & Chaudhari, K.S. (1992):** An Economic Order Quantity Model for Items with Two Level of Shortage for Linear Trend in Demand. J.Opt. Res. Soc., 43 (2), 157-167.
6. **Hari Kishan & Mishra, P.N. (1995):** Inventory Model with Stock Dependent Demand Rate and Constant Rate of Deterioration. Acta Ciencia Indica, XXI (4M), 420-422.
7. **Hari Kishan & Mishra, P.N. (1997):** An Inventory Model with Exponential Demand and Constant Rate of Deterioration with Shortage. Ind. Jour. Math. 37, 275-279.
8. **Hari Kishan, Megha Rani and Deep Shikha (2012):** Inventory model of deteriorating products with life time under declining demand and permissible delay in payment. Aryabhatta Journal of Mathematics and Informatics. 4(2), 307-314.
9. **Kar, S. Bhunia, A.K. and Maiti, M. (2001):** Deterministic Inventory Model with Two Level of Storage, a Linear Trend in Demand Rate and a Fixed Time Horizon. Comp. Oper. Res. 28, 1315-1331.
10. **Khanra, S, Sana, S.S. and Chaudhuri, K.S. (2010):** An EOQ model for perishable item with stock and price dependent rate, International Journal of Mathematics in Operational Research,2(3)320-335.
11. **Kundu, A. Chakrabarty, P. and Chakrabarty, T. (2013):** An EOQ model for items with alternating demand rates allowing shortages by considering time value of money. Yug. Jour. Oper. Res. 23(2), 263-278.
12. **Mandal, B.N. & Phaujdar (1989):** An Inventory Model for Deteriorating Items and Stock-dependent Consumption Rate. Jour. Oper. Res. Soc. 40 (5), 483-486.
13. **Mandal, B. and Pal, A.K. (1998):** Order level inventory model with ramp type demand rate for deteriorating items", J. of Interdisciplinary Mathematics, 1, 49-66.
14. **Mishra, S.S., Pandey, H. and Balram (2006):** A Probabilistic Inventory Model for Deteriorating Items with Non-linear Production Rate and Uniform Declining Demand Rate. Int. Jour. Agri. Stat. Sci., 2, 111-120.
15. **Sanjay Jain & Mukesh Kumar (2007):** An inventory model with inventory level dependent demand rate, shortages and decrease in demand. Jour. Comb. Inf. Sys. Sci. 32, 93-105
16. .**Santosh Kumar U. and Pakkala, T. P. M. (2001):** A stochastic inventory model for deteriorating items with random supply quantity, Opsearch, 38, 283-298.