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Optimal Ordering Decision for Decaying Items with Season Pattern Demand and Lead Time

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ABSTRCT

In this chapter shortage is allowed, lead-time is considered and seasonal pattern demand is taken in consideration. Here we discuss a model in which the retailer's and supplier's ordering policy is a function of deterioration, capital constraint, supplier's uncertain lead time, expiration date of product and the retailer's seasonal pattern demand. Here, we developed a deteriorating inventory model for the products which deteriorate very fast and presented an algorithm to derive the supplier's managing cost, economic order quantity, shortage period and the retailer's optimal replenishment cycle.

INTRODUCTION

Supply chain management encompasses the planning and management of all activities involved in sourcing, procurement, conversion, and management. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, Supply Chain Management integrates supply and demand management within and across companies. Supply Chain Management is the process of planning, implementing, and controlling the operations of the supply chain with the purpose to satisfy customer requirements as efficiently as possible. Supply chain management spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point-of-origin to point-of-consumption. The effective management of supply channel inventories is perhaps the most fundamental objective of SCM. Manufacturers procure raw material and process them in to finished goods, and sell the finished goods to distributors, then to retailer and/or customer. Within each organization the supply chain includes all functions involved in receiving and fulfilling a customer request. These functions include new product development, marketing, operations, distribution, and finance and customer service. The objective of every supply chain should be to maximize the overall value generated. For most supply chains the value would be strongly correlated with the overall profitability of the supply chain, which is calculated as the difference between the revenue generated from the customer and the overall cost across the supply chain. For a sustainable supply chain, it is very important that the only source of revenue in the whole supply chain, the customer, be given due attention. All other cash flows are simply fund exchanges that occur within the supply chain, given that different stages have different owners. All flows of information, products or funds generate costs within the supply chain. Thus the appropriate management of these flows is a key to supply chain success. Effective supply chain management involves the management of supply chain assets and products, information and fund flows to maximize the total supply chain profitability.

The supply chain models in inventory are a comparatively new for researchers. There has been especially very limited research for a supply chain. The idea of joint total cost of the supplier and the customer was first introduced by **Goyal** (1976). Later, **Cohen and Lee** (1988) determined material requirement for all materials at every stage in a supply chain. **Pake and Cohen** (1993) extended the above study to consider for stochastic sub systems. **Gyana and Bhabha** (1999) explored a single manufacturing system for procurement of raw materials with a multi-ordering policy that minimizes the total inventory costs of both the raw materials and the finished goods. **Sarker et.al** (2000) explored a supply chain model for determining an optimal ordering policy under allowable shortages. **Chien and Lin** (2004) investigated the optimal order interval and discount price such that the joint total cost is minimized during a finite planning horizon. **Ahmed et. al** (2007) have coordinated a two level supply chain in which they considered production interruptions for restoring of the quality of the production process.

The uncertainty in the lead time of a supplier is such a phenomenon which has deep roots in reality. Almost every supplier faces this problem at some time or the other during his business deals. **Liao and Shyu** (1991) showed that lead time can be controlled through crashing. Later on, **Ouyang et. al.** (1999) investigated uncertain lead time and studied the effect of cost reduction in a model. Even then, this is an area which has not been sufficiently explored by researchers.

Another area which is comparatively untouched is the concept of capital constraint for the supplier. However, as is very much evident from the face of facts, this constraint is very common. **Khouja and Mehrez** (1996) explored a constrained multi product newsboy problem with a progressive multiple discount. **Lau and Lau** (1996) in the same year studied the newsstand problem for a capacitated multi product single period inventory system. **Khouja** (1999) studied the newsboy problem by classifying it into two different categories, one with multi products under capital constraint, and the other with different supplier pricing policies. Lately, **Pasternack** (2001) has put forth his study on the newsboy problem with revenue sharing.

In the present study, we have strived to combine all the above mentioned factors into a single problem. We shall undertake to explore a three echelon supply chain, comprising of a supplier, a retailer and end customers. The supplier experiences the problem of lead time at his end. The lead time of the supplier is a probability density function of his managing cost. The more the supplier is ready to spend as managing expenses, smaller will be the lead time and vice versa. The product starts deteriorating as soon as they reach the retailer, wherefore their demand also decreases with decay. There is even an expiration date beyond which the product cannot be used. Hence, the retailer plans out his cycle to finish off his inventory before the product reaches its expiration date. As a matter of fact, the retailer takes the length of the lifetime of the product to be his planning horizon and does not wish to retain inventory after that date. We discuss model with variable deterioration rate for the products, which deteriorate very fast. In the formulation of inventory models, there are two subjects of consideration, one is the deterioration of items and other is the variation in demand. Under the first consideration most of the researches work on the problem in which the deterioration rate is taken as a constant but in most of the cases it rarely happens since the items such as food products, batteries, chemicals and photographic films etc. deteriorate very fast. In this chapter shortage is allowed, lead-time is considered and seasonal pattern demand is taken in consideration. The following notations and assumptions are applied in development of the model.

ASSUMPTIONS

1. The warehouses has unlimited capacity.
2. Supplier's delivery cost is not considered.
3. The lead-time do not exceed the seasonal interval.
4. The deteriorated items will not be replaced or repaired during a given cycle.
5. Backlogged demand is satisfied at the beginning of each replenishment.
6. Due to impatient customers, demand during the shortage period is partially lost.
7. In η is the waiting time then the fraction of customers backordered is given by: -

$$\theta(\eta) = 1 - \frac{\eta}{T} \qquad 0 \leq \eta < T$$

8. The retailer's selling price per unit p and the retailer's selling price per unit when shortage occur then p_b is such that : -

p_b = □ p > c where 0 < □ < 1

9. Seasonal pattern demand for the product follows a deterministic function of price and season such that : -

$$d_j(t,p) = \begin{cases} \dfrac{\alpha w(j)}{p^{\beta}} & j = 1,2,..........N \\ 0 & j > N \end{cases}$$

and w(j) is given by : -

$$w(j) = \frac{N - j + 1}{N}$$

NOTATIONS

p	retailer's selling price per unit
p_b	retailer's selling price per unit when shortage occur
□ (η)	the fraction of customer's backordered with the condition that they receive their order after η units of time
T	length of seasonal interval
Q	retailer's order quantity for each replenishment
Q_1	retailer's sales amount without backordering over the replenishment cycle.
Q_2	retailer's backorder quantity at the end of the replenishment cycle
N	a discrete number
NT	expiration date of product
v	a critical time at which inventory level reaches zero in the last season.
ξ	Supplier's managing cost for reducing lead-time.
k	constant deterioration rate of on hand stock k > 0.
c	retailer's wholesale purchase price per unit.
c_m	supplier's production cost per unit, $c_m < c$
c_o	retailer's ordering cost per replenishment cycle.
h	retailer's unit inventory holding cost per unit time.
r	retailer's penalty cost per unit of a lost sale including loss of profit.
□	Processing cost including making an inventory and deteriorated item per season.
w	supplier's maximal capital constraint.
y	supplier's lead time i.e. a delivery time which depends on the supplier's managing cost. We assume that the higher managing cost will always results in a lower lead time.
$I_j(t)$	retailer's inventory level at t during the j^{th} season $0 \le t < T$
$I_s(t)$	Supplier's inventory level at t before the beginning of a cycle when the supplier completes the order early by y unit time, $y \le t \le 0$.
f(y)	probability density function of the supplier's lead time y.
	$F_R(n, v)$ retailer's unit time profit.
$E_s(n, v, \xi)$	supplier's expected unit time profit function.
kt	Deterioration rate, here k is a very small positive constant. 0<k<<1

We will discuss our problem using two cases: -

1. In which there is no capital constraint.

2. In which capital constraint is considered.

1. No Capital Constraint: -

a. When the supplier's lead time y ≤ 0: -

In this case, the supplier accomplishes the order earlier than needed by the buyer, thus the product will be held until it is not at the target delivery date, this will incur on extra inventory cost for the supplier. Suppose the retailer's replenishment cycle is set at nT. Let $I_n(t)$, $n \le N$, be the inventory level during the n^{th} season. The differential equations governing the transition of the system during the season interval are: -

$$\frac{d\,I_1(t)}{d\,t} = P - k\,t\,I_1(t) - \frac{\alpha\,w(1)}{p^{\beta}} \qquad 0 \le t \le T \qquad \text{….. (1)}$$

The solution of equation (1) Omitting the higher powers of k :is -

$$I_1(t)e^{\frac{k\,t^2}{2}} = [\;P - \frac{\alpha\,w(1)}{p^{\beta}}\;][\;t + \frac{k}{2}\frac{t^3}{3}\;] + c \qquad \text{….. (2)}$$

Using boundary condition $I_1(0) = 0$

$$I_1(t) = [\;P - \frac{\alpha\,w(1)}{p^{\beta}}\;](t + \frac{k\,t^3}{6})\;e^{\frac{-k\,t^2}{2}} \qquad \text{….. (3)}$$

Now let $I_2(t)$ be the inventory level for the manufacturer during 2^{nd} season:-

$$\frac{dI_2(t)}{d\,t}) = -k\,t\,I_2(t) - \frac{\alpha\,w(2)}{p^{\beta}} \qquad \text{….. (4)}$$

With boundary condition $I_2(0) = I_1(T)$

Solution of eq^n (4) will be:-

$$I_2(t)e^{\frac{k\,t^2}{2}} = -\frac{\alpha\,w(2)}{p^{\beta}}\;(t + \frac{k\,t^3}{6})\,d\,t + c \qquad \text{….. (5)}$$

Using boundary condition, $I_2(0) = c$

$$I_2(t) = -\frac{\alpha\,w(2)}{p^{\beta}}\;(t + \frac{k\,t^3}{6})\;e^{-\frac{k\,t^2}{2}} + [\;P - \frac{\alpha\,w(1)}{p^{\beta}}\;](T + \frac{k\,T^3}{6})\;e^{-\frac{k}{2}(T^2+t^2)} \qquad \text{….. (6)}$$

Now if $I_3(t)$ be the inventory level during 3^{rd} season:-

$$\frac{dI_3(t)}{d\,t}) = -k\,t\,I_3(t) - \frac{\alpha\,w(3)}{p^{\beta}} \qquad \text{….. (7)}$$

With boundary condition $I_3(0) = I_2(T)$

The solution of equation (7) using boundary condition:

$$I_3(t) = -\frac{\alpha\,w(3)}{p^{\beta}}\;(t + \frac{k\,t^3}{6})\;e^{-\frac{k\,t^2}{2}} + [\;-\frac{\alpha\,w(2)}{p^{\beta}}\;](T + \frac{k\,T^3}{6})\;e^{-\frac{k}{2}(T^2+t^2)} + [\;P - \frac{\alpha\,w(1)}{p^{\beta}}\;](T + \frac{k\,T^3}{6})e^{-\frac{k}{2}(2T^2+t^2)}$$

….. (8

If $I_4(t)$ be the inventory level during 4^{th} season:-

$$\frac{dI_4(t)}{d\,t} = -k\,t\,I_4(t) - \frac{\alpha\,w(4)}{p^{\beta}} \qquad \text{….. (9)}$$

with boundary condition, $I_4(0) = I_3(T)$,solution will be:-

$$I_4(t) = -\frac{\alpha\, w(4)}{p^{\beta}}\,(T+\frac{kT^3}{6})\, e^{-\frac{kT^2}{2}} + [\ -\frac{\alpha\, w(3)}{p^{\beta}}\ ]\,(T+\frac{kT^3}{6})\, e^{-\frac{k}{2}(T^2+t^2)}$$
….. (10)

$$+[\ -\frac{\alpha\, w(2)}{p^{\beta}}\ ]\,(T+\frac{kT^3}{6})e^{-\frac{k}{2}(2T^2+t^2)} + [\ P-\frac{\alpha\, w(1)}{p^{\beta}}\ ]\,(T+\frac{kT^3}{6})e^{-\frac{k}{2}(3T^2+t^2)}$$

Similarly the inventory level during jth season:-

$$I_j(t) = -\frac{\alpha\, w(j)}{p^{\beta}}\,(t+\frac{kt^3}{6})e^{-\frac{kt^2}{2}} + \sum_{\substack{l=2\\ m=l-j\\ j\ge 3}}^{j-1} [\ -\frac{\alpha\, w(l)}{p^{\beta}}\ ]\,(T+\frac{kT^3}{6})e^{\frac{k}{2}(mT^2-t^2)}$$

$$+[\ P-\frac{\alpha\, w(1)}{p^{\beta}}\ ]\,(T+\frac{kT^3}{6})e^{-\frac{k}{2}[\ (j-1)T^2+t^2\ ]}$$
….. (11)

Now the unit time profit for the manufacturer:-

$F_m(n,v) = \frac{1}{nT}[$ Sales revenue – production cost – deteriorated cost – inventory holding cost – managing cost – shortage cost $]$

$$F_m(n,v) = \frac{1}{nT}[\ R_m(n,v) - C_m - C_d - H(n,v) - \zeta_m - S\,c_m\ ]$$
….. (12)

Where $R_m(n,v) = p\,Q_{m1} + \lambda\, p\,Q_{m2}$ ….. (13)

$$Q_{m1} = \sum_{j=1}^{n-1}\int_0^T d_j(t,p)\,dt + \int_0^v d_n(t,p)\,dt$$

$$Q_{m1} = \frac{\alpha\,(n-1)(2N+2-n)T}{2N\,p^{\beta}} + \frac{\alpha\,(N+1-n)v}{N\,p^{\beta}}$$
….. (14)

Since the manufacturer willing to wait for backorders of new items during stockout, the demand is assumed to be $d_1(t,p)$:-

$$Q_{m2} = \int_v^T d_1(t,p)\ \theta(T-t)\,dt$$

$$Q_{m2} = \frac{\alpha}{p^{\beta}T}(\frac{T^2}{2}-\frac{v^2}{2})$$
….. (15)

The production quantity at each replenishment is:-

$Q_m = I_1(0) + Q_{m2}$

$C(n,v) = [\ I_1(0) + Q_{m2}\ ]\,c$ ….. (16)

The lost sale amount is:-

$$= \int_v^T d_1(t,p)[\ 1-\theta(T-t)\ ]\,dt$$

The cost of lost sale amount:-

$$L_m(n,v) = \frac{\alpha}{p^{\beta}}(\frac{T}{2}+\frac{v^2}{2T}-v)\ r$$
….. (17)

and $B_m(n,v) = n\,\mu$ ….. (18)

Shortage cost $= c_s \int_v^T I_n(t)\,dt$

During the stockout period:-

$$\frac{d\,I_n(t)}{d\,t} = -\frac{\alpha\, w(1)}{p^\beta} \qquad v \le t \le T$$

$$I_n(t) = \frac{\alpha\, w(1)}{p^\beta}(v - t)$$

Then shortage cost $= c_s \dfrac{\alpha}{p^\beta}(vT - \dfrac{T^2}{2} - \dfrac{v^2}{2})$ ….. (19)

Now handling cost

$$H_m(n,v) = H_1^T + \sum_{j=2}^{n-1} H_j^T + H_n^v \qquad n \ge 3 \qquad ….. (20)$$

$$H_1^T = h\int_0^T I_1(t)\,dt$$

$$= h\,[\; P - \frac{\alpha\, w(1)}{p^\beta} \;]\int_0^T (t + \frac{k\,t^3}{6})\,(1 - \frac{k\,t^2}{2} + ...)\;dt$$

Now omitting the higher powers of k:-

$$H_1^T = h\,[\; P - \frac{\alpha\, w(1)}{p^\beta} \;]\int_0^T (t + \frac{k\,t^3}{6})\,(1 - \frac{k\,t^2}{2})\,dt$$

$$H_1^T = h\,(P - \frac{\alpha}{p^\beta})\,[\; \frac{T^2}{2} - \frac{k\,T^4}{12} \;] \qquad ….. (21)$$

also

$$H_j^T = h\int_0^T I_j(t)\,dt$$

$$H_j^T = h\int_0^T [\; -\frac{\alpha\, w(j)}{p^\beta} \;](t + \frac{k}{6}t^3)\; e^{-\frac{k\,t^2}{2}}\,dt$$

$$+ h\sum_{\substack{l=2 \\ m=l-j \\ j\ge 3}}^{j-1} \int_0^T [\; -\frac{\alpha\, w(l)}{p^\beta} \;](T + \frac{k}{6}T^3)\; e^{\frac{k}{2}(mT^2 - t^2)}\,dt$$

$$+ h\int_0^T [\; P - \frac{\alpha\, w(l)}{p^\beta} \;](T + \frac{k}{6}T^3)\; e^{-\frac{k}{2}[\;(j-1)T^2 + t^2\;]}\,dt \qquad ….. (22)$$

Solving I term:-

I term $= h\,[\; -\dfrac{\alpha\, w(j)}{p^\beta} \;]\displaystyle\int_0^T (t + \frac{k}{6}t^3)(1 - \frac{k\,t^2}{2})\,dt$

I term $= h\,[\; -\dfrac{\alpha\, w(j)}{p^\beta} \;][\; \dfrac{T^2}{2} - \dfrac{k}{12}T^4 \;]$

Solving II term:-

$$\text{II term} = h\sum_{\substack{l=2\\ m=l-j\\ j\geq 3}}^{j-1}\left[\ -\frac{\alpha\, w(l)}{p^{\beta}}\ \right](T+\frac{k}{6}T^{3})\int_{0}^{T}\left[\ 1+\frac{k}{2}(mT^{2}-t^{2})\ \right]dt$$

$$\text{II term} = h\sum_{\substack{l=2\\ m=l-j\\ j\geq 3}}^{j-1}\left[\ -\frac{\alpha\, w(l)}{p^{\beta}}\ \right](T^{2}+\frac{k}{2}T^{4})$$

Solving III term:-

$$\text{III term} = h\left[\ P-\frac{\alpha}{p^{\beta}}\ \right]\left[\ T^{2}-\frac{k}{2}(j-1)T^{4}\ \right]$$

Put all these values in equation (22):-

$$H_{j}^{T} = h\left[\ -\frac{\alpha\, w(j)}{p^{\beta}}\ \right](\frac{T^{2}}{2}-\frac{k}{12}T^{4})+h\sum_{\substack{l=2\\ m=l-j\\ j\geq 3}}^{j-1}\left[\ -\frac{\alpha\, w(l)}{p^{\beta}}\ \right](T^{2}+\frac{k}{2}mT^{4})$$

$$+h(P-\frac{\alpha}{p^{\beta}})\left[\ T^{2}-\frac{k}{2}(j-1)T^{4}\ \right] \qquad \text{..... (23)}$$

$$H_{n}^{v} = h\int_{0}^{v} I_{n}(t)\,dt$$

In general

$$\frac{d\,I_{n}(t)}{d\,t} = -k\,t\,I_{n}(t)-\frac{\alpha\, w(1)}{p^{\beta}}$$

$$I_{n}(t)\,e^{\frac{k\,t^{2}}{2}} = -\frac{\alpha\, w(1)}{p^{\beta}}\left[\ t+\frac{k}{2}\frac{t^{3}}{3}\ \right]+c$$

Using boundary condition $I_{n}(v)=0$

$$I_{n}(t) = \frac{\alpha\, w(1)}{p^{\beta}}e^{-\frac{k\,t^{2}}{2}}\left[\ (v-t)+\frac{k}{6}(v^{3}-t^{3})\ \right]$$

and

$$H_{n}^{v} = h\frac{\alpha\, w(1)}{p^{\beta}}\int_{0}^{v}(1-\frac{k\,t^{2}}{2})\left[\ (v-t)+\frac{k}{6}(v^{3}-t^{3})\ \right]dt$$

$$H_{n}^{v} = \frac{h\alpha}{p^{\beta}}(\frac{v^{2}}{2}+\frac{k}{12}v^{4}) \qquad \text{..... (24)}$$

Now let $I_{s}(t)$ be the supplier's inventory level at t before the beginning of a cycle.

$$\frac{d\,I_{s}(t)}{d\,t} = -k\,t\,I_{s}(t)$$

with boundary condition $I_{s}(0)=I_{1}(0)$

$$\frac{d\,I_{s}(t)}{d\,t}+k\,t\,I_{s}(t)=0 \qquad \text{..... (25)}$$

Solution using boundary condition

$$I_s(0) = c = I_1(0)$$

$$I_s(t) = I_1(0)\, e^{-\frac{kt^2}{2}} \quad \text{….. (26)}$$

Now let $I_1(t)$ be the retailer's inventory level at t during Ist season:-

$$\frac{dI_1(t)}{dt} = -kt\, I_1(t) - \frac{\alpha\, w(1)}{p^\beta} \quad \text{….. (27)}$$

With initial condition $I_1(0) = S$, solution is given by

$$I_1(t) = -\frac{\alpha\, w(1)}{p^\beta}(t + \frac{kt^3}{6})\, e^{-\frac{kt^2}{2}} + S\, e^{-\frac{kt^2}{2}}\, dt \quad \text{….. (28)}$$

Now if I_2 (t) be the inventory level during IInd season:-

$$\frac{dI_2(t)}{dt} = -kt\, I_2(t) - \frac{\alpha\, w(2)}{p^\beta} \qquad 0 \le t \le T \quad \text{….. (29)}$$

With initial condition $I_2(0) = I_1(T)$, the solution is

$$I_2(t) = -\frac{\alpha\, w(2)}{p^\beta}(t + \frac{k}{6}t^3)e^{-\frac{kt^2}{2}} - \frac{\alpha\, w(1)}{p^\beta}(T + \frac{k}{6}T^3)\, e^{-\frac{k(T^2+t^2)}{2}+} + S\, e^{-\frac{k(T^2+t^2)}{2}}$$

$$I_2(t) = -\frac{\alpha\, w(2)}{p^\beta}(t + \frac{k}{6}t^3)e^{-\frac{kt^2}{2}} - \frac{\alpha\, w(1)}{p^\beta}(T + \frac{k}{6}T^3)\, e^{-\frac{k}{2}(T^2+t^2)} + S\, e^{-\frac{k}{2}(T^2+t^2)} \quad \text{. (30)}$$

Now if $I_3(t)$ be the inventory level during 3rd season:-

$$\frac{dI_3(t)}{dt} = -kt\, I_3(t) - \frac{\alpha\, w(3)}{p^\beta} \quad \text{….. (31)}$$

with boundary condition $I_3(0) = I_2(T)$, solution is

$$-\frac{\alpha\, w(1)}{p^\beta}(T + \frac{k}{6}T^3)e^{-\frac{kT^2}{2}(2T^2+t^2)} + S\, e^{-\frac{k}{2}(2T^2+t^2)} \quad \text{….. (32)}$$

If $I_4(t)$ be the inventory level during 4th season:-

$$\frac{dI_4(t)}{dt} = -kt\, I_4(t) - \frac{\alpha\, w(4)}{p^\beta} \quad \text{….. (33)}$$

solution will be:-

$$I_4(t)\, e^{\frac{kt^2}{2}} = -\frac{\alpha\, w(4)}{p^\beta}(t + \frac{k}{6}t^3) + c \quad \text{….. (34)}$$

$$I_4(t) = -\frac{\alpha\, w(4)}{p^\beta}(t + \frac{k}{6}t^3)e^{-\frac{kT^2}{2}} - \frac{\alpha\, w(3)}{p^\beta}(T + \frac{k}{6}T^3)e^{-\frac{k}{2}(T^2+t^2)}$$

$$-\frac{\alpha\, w(2)}{p^\beta}(T + \frac{k}{6}T^3)e^{-\frac{k}{2}(2T^2+t^2)} - \frac{\alpha\, w(1)}{p^\beta}(T + \frac{k}{6}T^3)e^{-\frac{k}{2}(3T^2+t^2)} + S\, e^{-\frac{k}{2}(3T^2+t^2)} \quad \text{.. (35)}$$

Similarly the inventory level during jth season:-

$$I_j(t) = -\frac{\alpha\, w(j)}{p^\beta}(t + \frac{k}{6}t^3)e^{-\frac{kt^2}{2}} - \sum_{\substack{l=2 \\ m=l-j \\ j\ge 3}}^{j-1} \frac{\alpha\, w(l)}{p^\beta}(T + \frac{k}{6}T^3)e^{-\frac{k}{2}(mT^2-t^2)}$$

$$-\frac{\alpha\, w(1)}{p^{\beta}}(T+\frac{k}{6}T^{3})e^{-\frac{k}{2}[\ (j-1)T^{2}+t^{2}\ ]}+S\,e^{-\frac{k}{2}[\ (j-1)\,T^{2}+t^{2}\ ]} \quad \text{….. (36)}$$

Now the replenishment cycle and shortage length are set at nT and (T-v) units of time respectively.
Now when $y \le 0$, the retailer's unit time profit without late delivery is:-

$F_R(n,v)=\frac{1}{nT}[$ Sales revenue – purchasing cost – lost sale cost – processing cost – inventory holding cost – ordering cost – shortage cost $]$

$$F_R(n,v)=\frac{1}{nT}[\ R_R(n,v)-C_R(n,v)-L_R(n,v)-B_R(n,v)-H_R(n,v)-C_{R0}-S\,c_R\ ]$$

$$n \le N \qquad 0<v\le T \quad \text{….. (37)}$$

In case the supplier suffers inventory holding cost until the target due to early delivery, the supplier's unit time profit with early delivery by y unit time $F_s(n,v,\zeta,y)$ is:-

$$F_s(n,v,\zeta,y)=\frac{1}{nT}[\ \{\ I_1(0)+Q_2\ \}(c-c_p)-\{\ I_s(y)-I_1(0)\ \}c_p-h\int_y^0 I_s(t)\,dt-\zeta\ ] \quad \text{….. (38)}$$

$$n \le N \quad 0<v\le T \quad y\le 0$$

c_p = purchasing cost per unit of supplier's
Now for the retailer:-

$$R_R(n,v)=p\,Q_{R1}+\lambda\, p\,Q_{R2}$$

$$Q_{R1}=\sum_{j=1}^{n-1}\int_0^T d_j(t,p)\,dt+\int_0^v d_n(t,p)\,dt$$

$$Q_{R1}=\frac{\alpha\,(n-1)(2N+2-n)T}{2N\,p^{\beta}}+\frac{\alpha\,(N+1-n)\,v}{N\,p^{\beta}} \quad \text{….. (39)}$$

Since the retailer is willing to wait for backorders of new items during stockout, the demand is assumed to be $d_1(t,b)$:-

$$Q_{R2}=\int_v^T d_1(t,p)\,\theta(T-t)\,dt=\frac{\alpha}{p^{\beta}T}(\frac{T^2}{2}-\frac{v^2}{2}) \quad \text{….. (40)}$$

The order quantity at each replenishment is:-

$$Q_R=I_1(0)+Q_{R2}$$

$$\frac{C_R(n,v)}{c_R}=I_1(0)+Q_{R2}$$

$$C_R(n,v)=[\ I_1(0)+Q_{R2}\ ]\,c_R \quad \text{….. (41)}$$

The lost sale amount $=\int_v^T d_1(t,p)[\ 1-\theta(T-t)\ ]\,dt$

$$=\frac{\alpha}{p^{\beta}}(\frac{T}{2}+\frac{v^2}{2T}-v)$$

The cost of lost sale amount:-

$$L_R(n,v)=\frac{\alpha}{p^{\beta}}\left(\frac{T}{2}+\frac{v^2}{2T}-v\right) r \qquad \text{..... (42)}$$

processing cost:-

$$B_R(n,v)=n\,\mu \qquad \text{..... (43)}$$

Holding cost:-

$$H_R(n,v)=H_1^T+\sum_{j=2}^{n-1} H_j^T+H_n^v \qquad n\geq 3$$

$$H_1^T=h\int_0^T I_1(t)\,dt$$

$$=h\int_0^T\left[\frac{-\alpha\, w(1)}{p^{\beta}}\left(t+\frac{k}{6}t^3\right)e^{-\frac{k\,t^2}{2}}+s\, e^{-\frac{k\,t^2}{2}}\right]dt$$

$$H_1^T=\frac{-h\alpha}{p^{\beta}}\left(\frac{T^2}{2}-\frac{kT^4}{12}\right)+h\,S\left(T-\frac{k\,t^3}{6}\right) \qquad \text{..... (44)}$$

$$H_j^T=h\int_0^T I_j(t)\,dt$$

$$H_j^T=\frac{-h\alpha\, w(j)}{p^{\beta}}\left(\frac{T^2}{2}-\frac{kT^4}{12}\right)-\sum_{\substack{l=j-1\\ m=l-j}}^{2}\frac{h\,\alpha\, w(l)}{p^{\beta}}\left(T^2+\frac{m\,k\,T^4}{2}\right)$$

$$-\frac{h\alpha}{p^{\beta}}\left(T^2-\frac{k}{2}\,jT^4+\frac{kT^4}{2}\right)+h\,S\left(T-\frac{k\,jT^3}{2}+\frac{kT^3}{3}\right) \qquad \text{..... (45)}$$

Now $H_n^v=h\int_0^v I_n(t)\,dt$

First we have to calculate $I_n(t)$:-

$$\frac{d\,I_n(t)}{d\,t}=-k\,tI_n(t)-\frac{\alpha\, w(n)}{p^{\beta}} \qquad 0\leq t\leq T \qquad \text{..... (46)}$$

Solution using boundary condition $I_n(v)=0$

$$I_n(t)=\frac{\alpha\, w(n)}{p^{\beta}}e^{-\frac{kt^2}{2}}\left[(v-t)+\frac{k}{6}(v^3-t^3)\right] \qquad \text{..... (47)}$$

$$H_n^v=h\int_0^v\frac{\alpha\, w(n)}{p^{\beta}}e^{-\frac{kt^2}{2}}\left[(v-t)+\frac{k}{6}(v^3-t^3)\right]dt$$

$$H_n^v=\frac{\alpha\, w(n)}{p^{\beta}}\left(\frac{v^2}{2}+\frac{k}{12}v^4\right) \qquad \text{..... (48)}$$

Shortage cost $=c_s\int_v^T I_n(t)\,dt$

During the shockout period the inventory will be:-

$$\frac{d\,I_n(t)}{d\,t} = -\frac{\alpha\, w(1)}{p^{\beta}} \qquad v \le t \le T$$

$$S.C = c_s \frac{\alpha\, w(1)}{p^{\beta}} [\; vT - \frac{T^2}{2} - \frac{v^2}{2} \;]$$

Now when the supplier's lead time y > 0:-

The retailer's unit time profit with late delivery by y unit time $F_{Rd_R}(n,v,y)$ is:-

$$F_{Rd_R}(n,v,y) = \frac{1}{nT}[\; R\,d_R(n,v,y) - C\,d_R(n,v,y) - L\,d_R(n,v,y) - B_R(n,v) - S\,c - H\,d_R(n,v,y) - C_{0R} \;]$$

$n \le N \qquad 0 < v \le T \qquad y > 0$ ….. 49)

Here $R\,d_R(n,v,y) = p\,Q_{R1d} + \lambda\, p\,Q_{R2}$

We assume that the retailer can not promise the backordered quantity for the customers due to the supplier's delay. Therefore, the shortage without backordering will occur during the time interval $[0, y\;]$

$$Q_{R1d} = \sum_{j=1}^{n-1}\int_0^T d_j(t,p)\,dt + \int_0^v d_n(t,p)\,dt - \int_0^y d_1(t,p)\,d$$

$$Q_{R1d} = \frac{\alpha T}{N\,p^{\beta}}\frac{(n-1)}{2}(2N - n + 2) + \frac{\alpha}{p^{\beta}}\frac{(N-n+1)}{N}v - \frac{\alpha\, y}{p^{\beta}} \qquad \text{….. 50)}$$

The order quantity at each replenishment is:-

$$Q_{Rd} = I_1(y) + Q_{R2}$$

$$\frac{C\,d(n,v,y)}{c} = I_1(y) + Q_{R2}$$

$$C\,d(n,v,y) = [\; I_1(y) + Q_{R2}\;]\,c \qquad \text{….. (51)}$$

The lost sale amount is:-

$$\int_0^y d_1(t,p)\,d\,t + \int_v^T [\;1 - \theta(T,t)\;]d\,t$$

$$= \frac{\alpha}{p^{\beta}}\,y + \frac{\alpha}{p^{\beta}}(\frac{T}{2} - v + \frac{v^2}{2T})$$

The cost of lost sale amount is:-

$$L\,d(n,v,y) = \frac{\alpha\, y}{p^{\beta}}\,r + \frac{\alpha}{p^{\beta}}(\frac{T}{2} - v + \frac{v^2}{2T})\;r \qquad \text{….. (52)}$$

Since there is a potential lose $F_R(n,v,y) - F_{Rd}(n,v,y)$ on the retailer due to the supplier's delay, the supplier has to compensate the retailer. Therefore the retailer's unit time profit is:-

$$F_{Rd}(n,v,y) + [\; F_R(n,v) - F_{Rd}(n,v,y)\;] = F_R(n,v) \qquad \text{….. (54)}$$

i.e the retailer maintains his profit regardless of the supplier's unit time profit with late delivery by y unit time $F_{sd}(n,v,\zeta,y)$

$$F_{sd}(n,v,\zeta,y) = \frac{1}{nT}[\;\text{Sales revenue – deteriorated cost – compensated cost – managing cost}\;]$$

$n \le N \qquad 0 < v \le T \qquad y > 0$ ….. (55)

From equation (38) and (55) the supplier's expected unit time profit $E_s(n,v,\zeta)$ is:-

$$E_s(n,v,\zeta)=\int_{-\infty}^{0} F_s(n,v,\zeta,y)\, f(y)\, d\,y+\int_{0}^{\infty} F_{sd}(n,v,\zeta,y)\, f(y)\, d\,y \quad \text{….. (56)}$$

If the retailer determines the order quantity independently then the problem can be formulated as:-

Max : $F_R(n,v)$

S.t : $1\le n\le N,\qquad 0<v\le T$ ….. (57)

Now when supplier's capital constraint is considered

If the supplier's lead time $y\le 0$, the supplier's cost is:-

$G(n,v,\zeta,y)=[$ Production cost + Inventory holding cost + Deteriorated cost + Managing cost $]$ ….. (58)

If the supplier's lead time y > 0, the supplier's cost is-

$G\,d(n,v,\zeta,y)=[$ Production cost + Compensated cost + Deteriorated cost + Managing cost $]$ ….. (59)

From these equations, the supplier's expected cost is:-

$$E_G(n,v,\zeta)=\int_{-\infty}^{0} G(n,v,\zeta,y)\, f(y)\, d\,y+\int_{0}^{\infty} G\,d(n,v,\zeta,y)\, f(y)\, d\,y \quad \text{….. (60)}$$

$$n\le N \qquad 0<v\le T \qquad \zeta\ge 0$$

The supplier's optimization problem can be formulated as:-

Max : $E_s(n_R^*, v_R^*(n_R), \zeta)$ ….. (61)

S.t : $E_G(n_R^*, v_R^*(n_R), \zeta)\le w$

CONCLUSION: -

In the present chapter we discuss a model in which the retailer's and supplier's ordering policy is a function of deterioration, capital constraint, supplier's uncertain lead time, expiration date of product and the retailer's seasonal pattern demand. Here, we developed a deteriorating inventory model for the products which deteriorate very fast and presented an algorithm to derive the supplier's managing cost, economic order quantity, shortage period and the retailer's optimal replenishment cycle.

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