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LMI Based Robust H_∞ Stabilisation of Non-Holonomic Wheeled Mobile Robot with Time Delay

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ABSTRACT– Control of time delay systems has been a challenging problem for the control engineers. The issues have been inviting attention of many researchers, especially in the context of robust control. The presence of delay in the systems makes closed-loop stabilization difficult and degrades tracking performance. This paper is concerned with the robust H_∞control of a non-holonomic wheeled mobile robot with time delay by LMI approach. Also this paper gives a linearized model of the wheeled mobile robot (WMR) in the presence of disturbances, uncertainties and delays. A WMR is a wheeled vehicle which is capable of autonomous motion and is extensively used in lots of dangerous and heavy jobs, including the transportation of nuclear waste, mines, lunar exploration etc. A robust controller was synthesized for this system using LMI algorithms. It can be seen that, as the value of the of time delay increases, settling time of a WMR also increases but the steady state error remains constant. Also it can infer that, for larger values of delays the difference between the settling time and the delay value remains almost constant Simulation results show that the WMR system is stable with the proposed LMI H_∞ controller.

Keywords: H_∞ control, LMI control, robust control, time delay system, WMR.

INTRODUCTION

Wheeled Mobile Robot is a multi-input multi output system having interesting importance in both scientific research and practical applications. One of the promising applications is extended for the assistance of disabled, handicapped or elderly people. In control field, WMR researchers have focused on establishing mathematical modeling as well as control aspects such as trajectory tracking and stabilization. Unscented Kalman Filter (UKF) arithmetic in mobile robot control has been applied in [1] and gained better control effect, but its complexity limits its practical application. Ref[2]-[3] introduced intelligent arithmetic in this field and avoided establishing the precise mathematical models. But the design of control rules are highly dependent on personal experience, and its preciseness and stability are not satisfactorily addressed. In [4]-[5] research in WMR has been focused on tracking control, but the effect of disturbances and model uncertainties were not properly dealt with. In [6], a local feedback H_∞ robust controller has been designed for a wheeled mobile robot with time delay but the model uncertainties and the effect of disturbances were not taken into account. Moreover the effect of time delay with different types of uncertain disturbance were not discussed. A robust LMI H_∞ controller for WMR has been synthesized in [7] for a wheeled mobile robot having uncertainties in the disturbance, but the time delay was not considered. Since the main emphasize of this paper is on the control issues of uncertain time delay system, a robust LMI H_∞ controller is designed for a WMR system considering both time delay and model uncertainty simultaneously.

WMR is a complex unstable nonlinear coupled system with model uncertainties and time delay. This time delay is due to the position signal from robot to sensor and the execute signal from actor to robot. AS-R3-Degree of freedom differential WMR is a platform where new stabilization on robotic field can be tested. In this paper based on this platform, a linearized mathematical model with uncertainty and time delay is derived. The low frequency disturbances are assumed to act on the state of the system. Then, based on LMI, a robust H_∞ feedback controller is designed with first and second order uncertain disturbance. Finally, simulation is performed and the result shows the efficiency of the method.

The paper is organized as follows: Section II deals with the modeling of WMR in states pace form. Section III discusses the design methodology and the performance index calculation. Section IV gives the WMR H_∞ control scheme which include time delay and uncertainties. Section V is devoted to present numerical examples. Section VI gives the simulation results and the analysis of it. Finally Section VII concludes the paper, followed by the references used.

2. MODELING OF WMR

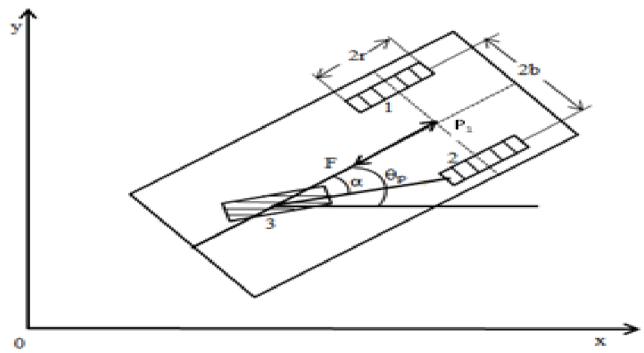


Fig.1 Geometrical model of the wheeled mobile robot

Fig.1 shows a schematic of the WMR in the X-Y coordinate axis. It consists of three modules, including drive module, control module, and sensor module which are connected and fixed through bolts. The robot keeps its balance by three wheels: two front wheels (1-2) are driving wheels and one rear wheel (3) is the steering wheel. The driving wheels are driven by two independent DC motors which act as the actuators. $F$is the projection of mass center of the robot.$P_1$is the centre of two front wheels of the robot. l_f is the distance between point P_1 and point F. $\theta_P$is the heading angle of the robot. $v_p$is the speed at P_1. $\omega$is the angular speed of WMR. $v_1$and $v_2$are the speed of left and right driving wheels respectively . $\alpha$is the angle of the steering wheel with the robot .This α will appear in the model as a steering command applied . The forward and rotational motions of WMR are facilitated by the control of 1 and 2.$v_p$can be written in terms of v_1,v_2 as $v_p = \frac{v_1+v_2}{2}$. Similarly ω can be expressed as$\omega = \frac{v_2-v_1}{2b}$

Then ω =$\dot{\theta}_p = v_p \sin\alpha$

Let (x_P,y_p) be the coordinates at P_1 and (x_f,y_f) the coordinates at F at a given point in time t. When the robot is moving at a particular speed v_p, the x and y axis speed at P_1can be expressed as:

$$\dot{x}_p = v_p cos\theta_p$$
$$\dot{y}_p = v_p sin\theta_p \tag{1}$$

The acceleration variables obtained from Eqn(1) are written as :

$$\ddot{x}_p = \dot{v}_p cos\theta_p - \dot{y}_p \omega$$
$$\ddot{y}_p = \dot{v}_p sin\theta_p + \dot{x}_p \omega \tag{2}$$

Now the relation between$P_1$and F is

$$x_f = x_p - l_f cos\theta_p \tag{3}$$

$$y_f = y_p - l_f sin\theta_p$$

The acceleration variables obtained from Eqn(3) are written as:

$$\ddot{x}_f = \ddot{x}_p + l_f\omega^2 cos\theta_p + l_f\dot{\omega} sin\theta_p$$
$$\ddot{y}_f = \ddot{y}_p + l_f\omega^2 sin\theta_p - l_f\dot{\omega} cos\theta_p \quad (4)$$

Substituting Eqn(2) in Eqn(4)

$$\ddot{x}_f = \dot{v}_p cos\theta_p - \dot{y}_p\omega + l_f\dot{\omega} sin\theta_p + l_f\omega^2 cos\theta_p$$
$$\ddot{y}_f = \dot{v}_p sin\theta_p + \dot{x}_p\omega - l_f\dot{\omega} cos\theta_p + l_f\omega^2 sin\theta_p \quad (5)$$

The above relations denoted the kinematics of the system. Now to arrive at the dynamics of the WMR, let T_1 and T_2 be the driving torques on left and right front wheels of the WMR. The accelerations of the system depend not only on the inputs but are acted upon by disturbance. Let u=[u_1 $u_2]^T$ be the control input vectors. Uncertain disturbance vector $w = [w_1\ w_2]^T$denotes the disturbance put on the v_p andω.

Then $\dot{v}_p = \beta_1 u_1 + \beta_3 w_1 \dot{\omega} = \beta_2 u_2 + \beta_4 w_2$

where $\beta_1 = \frac{1}{Mr}$, $\beta_2 = \frac{b}{Jr}$, $\beta_3 = \frac{1}{M}$, $\beta_4 = \frac{1}{J}$ with r representing the radius of the wheel, M the mass of the WMR , J the moment of inertia with respect to the centre of mass F and $2b$ the length of the wheel axis.

Substituting the above expressions in Eqn (5) which yields

$$\ddot{x}_f = \beta_1 u_1 \cos\theta_p + \beta_3 w_1 cos\theta_p - \dot{y}_P\omega + l_f\beta_2 u_2 sin\theta_p + l_f\beta_4 w_2 sin\theta_p + l_f\omega^2 cos\theta_p$$
$$\ddot{y}_f = \beta_1 u_1 sin\theta_p + \beta_3 w_1 sin\theta_p + \dot{x}_p\omega - l_f\beta_2 u_2 cos\theta_p - l_f\beta_4 w_2 cos\theta_p + l_f\omega^2 sin\theta_p \quad (6)$$

To accommodate the time delay of the system, a matrix A_d is introduced in the system. Causes of Time delay in WMR system are

(a) In local controller, position signal from robot to sensor and the execute signal from actor to robot cause time delay.

(b)In remote network controller, the signal transmission causes time delay.

The state variables of the system are taken as $x = (\theta_p\ \dot{x}_f\ \dot{y}_f\ \dot{\theta}_p)^T$ which are angular displacement, x axis Speed at F, y axis speed at F and angular speed. Table 1.shows the nominal values of the WMR parameters [6].

Table 1.Parameters of the WMR system

Parameter	**Nominal Value**
l_f	0.09 m
r	0.105 m
M	25 kg
J	0.5512 kgm^2
2b	0.41 m

Linearizing the model around $\dot{y}_f = 0.01$and $\dot{\theta}_p = 0.0175$, the state space representation of the nominal open loop system is in the form

$$\dot{x} = A_1 x + A_d x(t-\tau) + B_1 w + B_1 u$$
$$y = C_1 x \qquad (7)$$

Where $A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -0.0175 & -0.01 \\ 0 & 0.0175 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $A_d = \begin{bmatrix} 0.3 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0 & 0 \\ 0.1 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}$

$B_1 = \begin{bmatrix} 0 & 0 \\ 0.04 & 0 \\ 0 & -0.163 \\ 0 & 1.814 \end{bmatrix}$ $B_2 = \begin{bmatrix} 0 & 0 \\ 0.381 & 0 \\ 0 & -0.319 \\ 0 & 3.542 \end{bmatrix}$ $C_1 = \begin{bmatrix} 0.01 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The nominal open loop system is unstable with all the poles on the imaginary axis.

3. DESIGN METHODOLOGY

For the above WMR system a robust controller has to be designed using LMI algorithms. A Lyapunov function candidate has to be selected for the construction of the linear matrix inequalities.

A Lyapunov function candidate for the system represented by Eqn(7) is given by

$$V(x,t) = x(t)^T P x(t) + \int_{t-\tau}^{t} x(\sigma)^T Q x(\sigma) d\sigma \qquad (8)$$

where $P, Q \in R^{nxn}$ are positive definite symmetric matrices [8].

If $P > 0,\ Q > 0$ satisfies$\dot{V}(x,t) < 0$for every x satisfying Eqn(7), then the system (7) is stable, i.e., $x(t) \to 0$ as $t \to \infty$

Let $u(t) = Kx(t)$, then the resulting closed loop system is

$$\dot{x} = (A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t) \qquad (9)$$

The time derivative of $V(x,t)$along the trajectory of the system represented by Eqn(9) is given by

$$L(x,t) = \dot{V}(x,t)$$
$$= x(t)^T P[(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)]$$
$$+[(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)]^T P x(t) + x(t)^T Q x(t) - x(t-\tau)^T Q x(t-\tau)$$

Introduce the following performance measure for disturbance attenuation

$$J = \int_0^\infty [z^T(t) z(t) - \gamma^2 w^T(t) w(t)] dt$$

This can be rewritten as

$$J \le [\int_0^\infty (z^T(t) z(t) - \gamma^2 w^T(t) w(t) + L(x,t)] dt$$

Substituting $z(t)$ and $L(x,t)$ in the above Eqn it can be written as

$$J \le \int_0^\infty \{[Cx(t)]^T [Cx(t)] - \gamma^2 w^T(t) w(t) + x(t)^T P[(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)]$$
$$+ [(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)]^T P x(t) + x(t)^T Q x(t) - x(t-\tau)^T Q x(t-\tau)\} dt$$

From this performance measure inequality, the linear matrix inequality for the WMR system can be expressed as:

$$\begin{bmatrix} (A_1X+B_2Y)^T+A_1X+B_2Y+Q & B_1 & (C_1X)^T & A_dX \\ B_1^T & -\gamma I & 0 & 0 \\ C_1X & 0 & -\gamma I & 0 \\ XA_d^T & 0 & 0 & -Q \end{bmatrix} < 0 \qquad (10)$$

$$X > 0$$

where $X(=X^T)$, Q and Y are the matrices and γ is the H_∞ performance attenuation bound. Using LMI tool box in MATLAB® we can get suited matrix X and Y by the MATLAB code X=dec2mat(lmis,xfeas,x) and Y=dec2mat(lmis,xfeas,y). Then a state feedback robust H_∞ controller $u = YX^{-1}x(t)$ can be obtained to guarantee the stability of the system [9].

4. WMR H∞ CONTROL SCHEME

The H_∞ control scheme for the WMR is shown in Fig. 2 which depicts the output feedback structure of the uncertain plant with time delay .In order to get a robust controller, the modeling errors, parameter uncertainties etc. are also to be considered. For the WMR system, the robot's parameters are subjected to variations and hence uncertainties arise in the system modeling. To accommodate the parameter variations and nonlinearities w is acted on by unstructured weighting W_W.

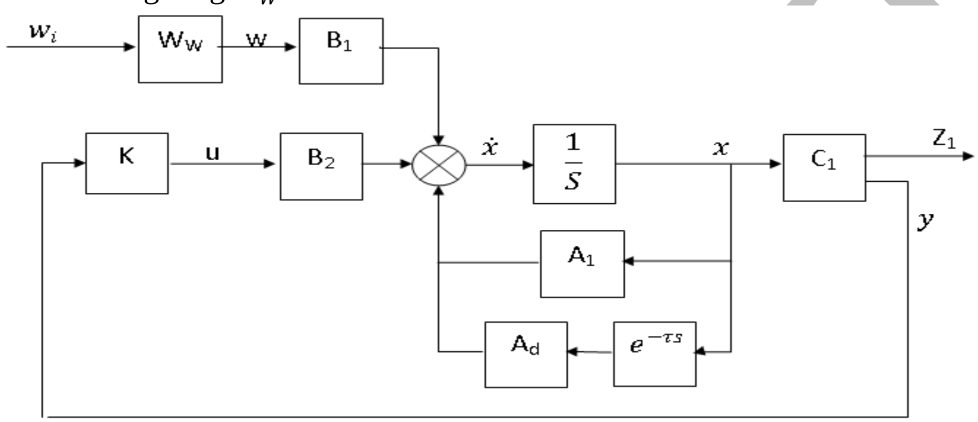


Fig 2 Augmented structure of uncertain plant with time delay.

Let w_i be a two input disturbance assumed to be acting on the state. These disturbances on the plant are by nature of low frequency. Let W_W be the low frequency disturbance weighting for the input w_i. Then $w = W_W w_i$
The W_W can be represented as

$$W_W = C_W(sI - A_W)^{-1}B_W + D_W \qquad (11)$$

Then the state space matrices of the transfer function $[A_W, B_W, C_W, D_W]$ can be obtained.
Now consider the regulated or controlled output z_1
Then $z_1 = \varphi x$ where the regulated matrix corresponding to the state can be selected as $\varphi = C_1$
Let x_w correspond to the states of the low frequency disturbance of (11).
Then

$$\dot{x}_w = A_w x_w + B_w w_i$$
$$w = C_w x_w + D_w w_i \qquad (12)$$

Substituting Eqn (12) in Eqn (7) we get

$$\dot{x} = A_1x + A_dx(t-\tau) + B_1(C_w x_w + D_w w_i) + B_2u$$

$$\dot{x}_w = A_w x_w + B_w w_i$$

The augmented state space for the uncertain plant is as in (13)

$$\begin{bmatrix} \dot{x} \\ \dot{x}_w \end{bmatrix} = \begin{bmatrix} A_1 & B_1 C_W \\ 0 & A_W \end{bmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + \begin{bmatrix} B_1 D_W & B_2 \\ B_W & 0 \end{bmatrix} \begin{bmatrix} w_i \\ u \end{bmatrix} + A_d x(t-\tau)$$
$$z_1 = \varphi x \qquad (13)$$

5. NUMERICAL EXAMPLES

As mentioned earlier, an LMI H_∞ controller is synthesized for the WMR system with time delay. Here we are considering two cases. In the first case we are designing a robust controller for the system when the unstructured uncertainty weighting W_W is a first order Transfer function while in the second case a second order Transfer function W_W is applied to it.

Case 1 when the unstructured uncertainty weighting W_W is a first order Transfer function

Let $W_W = \frac{400}{S+400} I = C_W (sI - A_W)^{-1} B_W + D_W$ and the values of $A_1, A_d, B_1 \& B_2$ are as given by Eqn (7) .Then the state space matrices of the transfer function $[A_W, B_W, C_W, D_W]$ can be obtained.

Now an H- infinity controller is synthesized in MATLAB® with the help of LMI solvers using Eqn (10). The closed loop responses of the system for different values of delay at optimum value of disturbance attenuation bound γopt were plotted.

Case 2 when the unstructured uncertainty weighting W_W is a second order Transferfunction

Let $W_W = \frac{400}{s^2+20s+400} I = C_W (sI - A_W)^{-1} B_W + D_W$

As in the previous case here also an H- infinity controller is synthesized in MATLAB® with the help of LMI solvers.

6.SIMULATION RESULTS AND DISCUSSION

The robust controller is synthesized inMATLAB® with the help of LMI solvers. Fig 3 shows the closed loop response of the given WMR system with $W_W = \frac{400}{S+400} I$ and the delay 0 sec. From this figure it can be seen that the given WMR system is stabilized by the proposed LMI H_∞ controller. Here the optimum value of gamma is found to be 0.146. Fig 4 depicts the closed loop response of the above system with different values of delays say 0.5 sec,1 sec,3sec and 5 sec respectively.

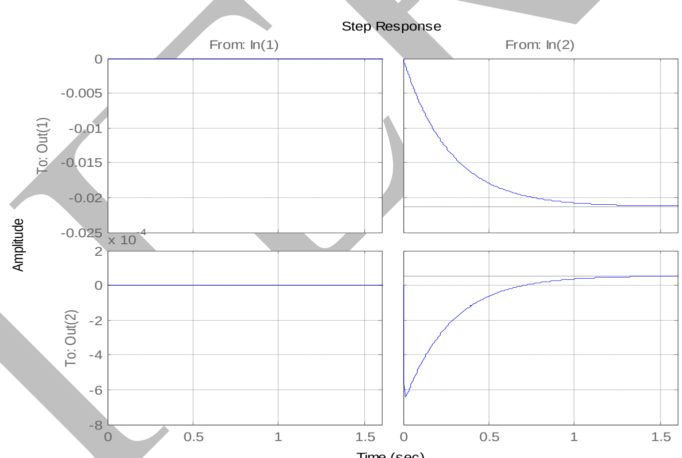


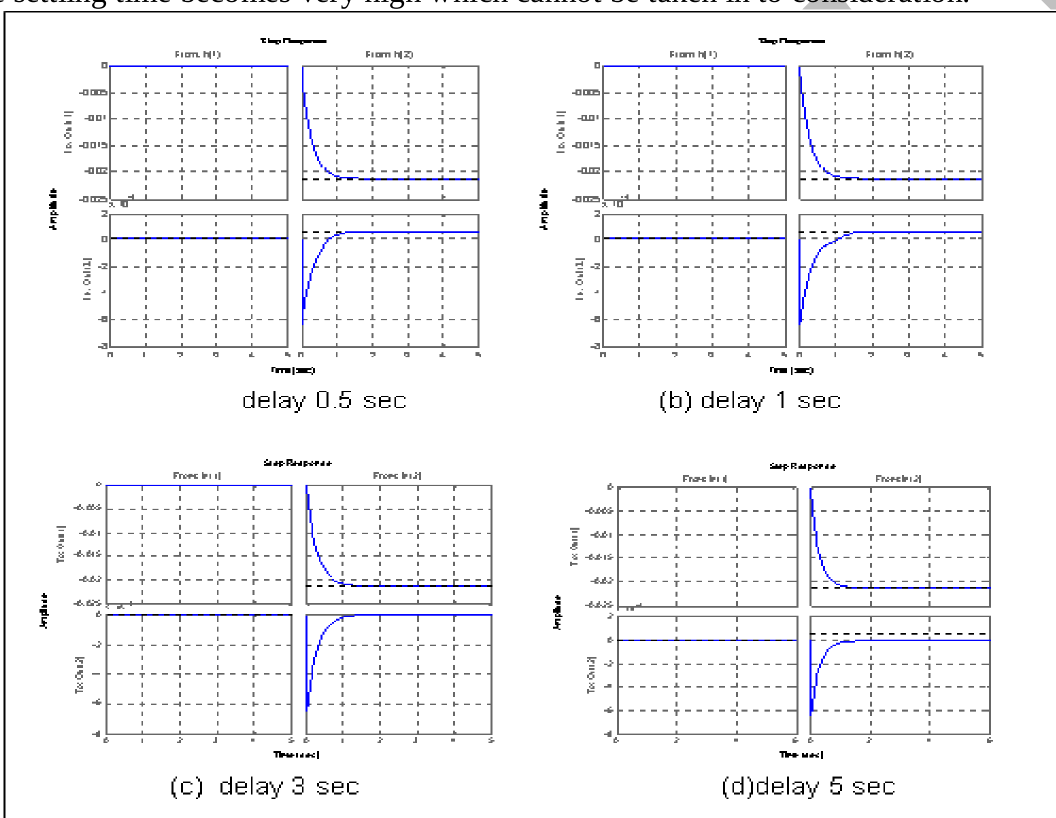
Fig.3 Closed loop response of the given WMR system with $W_W = \frac{400}{S+400} I$ and delay 0 sec Here also it can be seen that the WMR system is stable with the proposed H_∞ controller. Table 2 gives the Performance specification of

the above WMR system with $W_W = \frac{400}{S+400} I$ with optimum value of disturbance attenuation bound at different values of time delay

Table 2: Performance specification of the WMR system with $W_W = \frac{400}{S+400} I$ having different values of time delay.

Delay (sec)	γ opt=0.146	
	Settling time(sec)	Steady state error
0	1.09	-0.0213&0
0.1	1.1	-0.0213&0
0.5	1.18	-0.0213&0
1	1.46	-0.0213&0
2	2.41	-0.0213&0
3	3.4	-0.0213&0
4	4.4	-0.0213&0
5	5.4	-0.0213&0

From the above table it can be seen that, as the value of time delay increases, settling time also increases. Again it can be seen that whatever be the value of time delay the steady state error remains constant. Also we can infer that, for larger values of delays the difference between the settling time and the delay value remains almost constant. Here the maximum value of time delay is taken as 5 sec because when the delay value is more than this value, the settling time becomes very high which cannot be taken in to consideration.



delay 0.5 sec (b) delay 1 sec

(c) delay 3 sec (d)delay 5 sec

Fig.4 Closed loop response of the given WMR system with $W_W = \frac{400}{S+400} I$ and delay a) 0 .5sec b)1sec c) 3sec d) 5sec

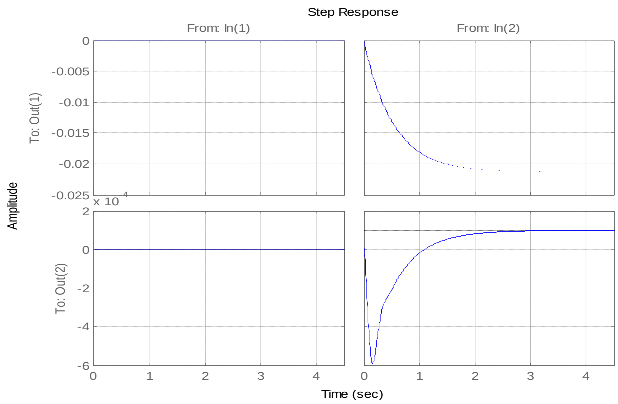


Fig 5: Closed loop response of the given WMR system with $W_W=\frac{400}{s^2+20s+400}I$ and delay 0 sec

Fig 5 shows the closed loop response of the given WMR system with $W_W=\frac{400}{s^2+20s+400}I$ and the delay 0 sec. As in the previous case here also it can be seen that the given WMR system is stabilized by the proposed LMI H_∞ controller. Here also the optimum value of disturbance attenuation bound is found to be 0.146. Table 3 gives the Performance specification of the above WMR system with $W_W=\frac{400}{s^2+20s+400}I$ having different values of time delay at optimum disturbance attenuation bound γopt=0.146.

Table 3: Performance specification of the WMR system with $W_W=\frac{400}{s^2+20s+400}I$ having different values of time delay

Time delay (s)	γ opt=0.146	
	Settling time(sec)	**Steady state error**
0	2.15	-0.0213&0
0.1	2.16	-0.0213&0
0.5	2.24	-0.0213&0
1	2.44	-0.0213&0
2	3.16	-0.0213&0
3	4.1	-0.0213&0
4	5.09	-0.0213&0
5	6.09	-0.0213&0

From the above table it can be see that, ,as the value of time delay increases , settling time also increases. Again it can be seen that whatever be the value of time delay the steady state error remains constant. Also we can infer that, for larger values of delays the difference between the settling time and the delay value remains almost constant. Hence It can be seen that even though the robust control of time delay systems are more complicated than that of the system without time delay, the proposed LMI H_∞ controller is successful in controlling the WMR with both delay and uncertain disturbances.

7. CONCLUSIONS

LMI control of WMR in the presence of disturbance as well as delay was discussed here. Also this paper gives a linearized model of the WMR in the presence of disturbances, uncertainties and delays. A robust controller was synthesized using LMI algorithms. The LMI solvers in MATLAB were made use of for solving the inequalities. It can be seen that, as the value of time delay increases, settling time also increases but the steadystate error remains constant. Also we can infer that for larger values of delays the difference between the settling time and the delay value remains almost constant. Simulation results show that the WMR system is stable with the proposed LMI H_∞ controller.

REFERENCES

1. Song Q, Qi J, and Han J (2006) An adaptive UKF algorithm and its application in mobile robot control, IEEE Int. Conf. Robot. Biomimetics, 1117 - 1122.
2. Stoian V, Ivanescu M, Stoian E, and Pana C (2006) Using artificial potential field methods and fuzzy logic for mobile robot control International. Power Electronics and Motion Control Conference,.Portoroz, Slovenia 385 – 389
3. Zhang M (1997) Neural network and fuzzy logic techniques based collision avoidance for a mobile robot, Robotic, 15:627-632.
4. Chen H, Miao M, HuWang and Zi-Xing(2009) Moving Horizon H infinity Tracking Control of Wheeled Mobile Robots with Actuator Saturation IEEE Transactions on Control Systems Technology, Vol. 17, No.2 March2009
5. Oriolo, De Luca and Vendittelli (2002) WMR Control Via Dynamic Feedback Linearization: Design, Implementation, and Experimental Validation IEEE Transactions on Control System Technology, Vol. 10, No. 6, Nov.2002
6. Jiang W (2008) Robust H∞ Controller Design for Wheeled Mobile Robot with Time-delay International Conference on Intelligent Computation Technology and Automation, Tianjin China
7. Mary A D (2011) Robust LMI Based Control of Wheeled Mobile Robots The 6th International Conference on Computer Science & Education (ICCSE 2011) August 3-5, 2011. SuperStar Virgo, Singapore
8. Boyd S, Ghaoui L, Feron E, and Balakrishnan V (1995) Linear Matrix Inequalities in System and Control Theory Society for Industrial and Applied Mathematics Philadelphia
9. Gahinet P, Nemirovski A, Lamb A.J and Chilali M, *LMIControlToolbox: for use with MATLAB*. The Mathworks, Inc.Natick, MA. 1995.

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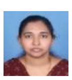

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