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Hall Effects on Steady MHD Flow over a Rotating Disk In Porous Medium With Heat Transfer

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Abstract:
The steady MHD flow in a porous medium of an incompressible viscous fluid above an infinite rotating porous disk is studied with the influence of Hall current and heat transfer. Numerical solutions of the nonlinear governing equations which govern the hydrodynamics and energy transfer are obtained. The effects of the MHD and Hall current on velocity and temperature distributions have been considered.

Keywords: rotating disk, porous medium, heat transfer, MHD, Hall current effect, numerical solution.

Introduction:
The study of magneto hydrodynamic flows with Hall currents has important engineering applications in problems of magneto hydrodynamic generators and of Hall accelerators as well as in flight magneto hydrodynamics. In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions; also, a current is induced in a direction normal to both the electric and magnetic fields. The phenomenon is called the Hall Effect.

Sherman and Sutton [1], Raptis and Ram [2], Sato [5], Pop [7], Hossain [8], Hossain and Mohammad [9], Hossain and Rashid [10], and Ram [11] studied the Hall effects. The influence of an external uniform magnetic field on the flow due to a rotating disk was studied [3, 4 and 12]. The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic flow was investigated [1, 5 and 8].

In the present work, the steady MHD flow of a viscous incompressible fluid due to the uniform rotation of a disk of infinite extent in a porous medium is studied with heattransfer and Hall effects. The flow in the porous media deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium. The temperature of the disk is maintained at a constant value. The governing nonlinear differential equations are integrated numerically using the finite difference approximations. The effects of the MHD and Hall current of the medium on the steady flow and heat transfer have been computed and discussed.

Basic equations:
Let the disk lie in the plane $z = 0$ and the space $z > 0$ is equipped by a viscous incompressible fluid. The motion is due to the rotation of an insulated disk of infinite extent about an axis perpendicular to its plane with constant angular speed ω through a porous medium where the Darcy model is assumed. Otherwise the fluid is at rest under pressure p_∞. The equations of steady motion are given by

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \quad (1)$$

$$\rho(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r})+\frac{\partial p}{\partial r}=\mu(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2})-\frac{\mu}{K}u -\sigma B_0{}^2 u-\frac{\sigma B_0{}^2(u+\phi v)}{(1+\phi^2)}, \quad (2)$$

$$\rho(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r})=\mu(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2})-\frac{\mu}{K}v -\sigma B_0{}^2 v-\frac{\sigma B_0{}^2(\phi u-v)}{(1+\phi^2)}, \quad (3)$$

$$\rho(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z})+\frac{\partial p}{\partial z}=\mu(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2})-\frac{\mu}{K}w, \quad (4)$$

where u, v, w are velocity components in the directions of increasing r, φ, z respectively, p is denoting the pressure, μ is the coefficient of viscosity, ρ is the density of the fluid, K is the Darcy permeability, σ electrical conductivity, B_0 is the applied magnetic field. The von Karman transformations [1],

$$u=r\omega F, \quad v=r\omega G, \quad w=\sqrt{\omega\nu}H, \quad z=\sqrt{\frac{\nu}{\omega}}\zeta, \quad p-p_\infty=-\rho\nu\omega P,$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of ζ and ν is the kinematic viscosity of the fluid, $\nu = \mu/\rho$. With these definitions, equations (1)–(4) take the form

$$\frac{dH}{d\zeta}+2F=0, \quad (5)$$

$$\frac{d^2F}{d\zeta^2}-H\frac{dF}{d\zeta}-F^2+G^2-MF-NF-\frac{N(F+\phi G)}{(1+\phi^2)}=0, \quad (6)$$

$$\frac{d^2G}{d\zeta^2}-H\frac{dG}{d\zeta}-2FG-MG-NG-\frac{N(\phi F-G)}{(1+\phi^2)}=0, \quad (7)$$

$$\frac{d^2H}{d\zeta^2}-H\frac{dH}{d\zeta}+\frac{dP}{d\zeta}-MH=0, \quad (8)$$

$M=\nu/K\omega$ is the porosity parameter and $N=\sigma B_0{}^2/\rho\omega$ is the magnetic parameter. $\phi=\omega_e t_e$ is Hall parameter. Where ω_e is the electron frequency. The boundary conditions for the velocity problem are given by

$$\zeta=0, \quad F=0, \quad G=1, \quad H=0, \quad (9a)$$

$$\zeta\to\infty, \quad F\to 0, \quad G\to 0, \quad P\to 0, \quad (9b)$$

Equation (9a) indicates the no-slip condition of viscous flow applied at the surface of the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in equation (9b). The above system of equations (5)–(7) with the prescribed boundary conditions given by equations (9) are sufficient to solve for the three components of the flow velocity. Equation (8) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid, heat Transfer takes place. The energy equation without the dissipation terms takes the form;

$$\rho c_p (u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}) - k(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}) = 0, \quad (10)$$

Where T the temperature of the fluid is, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞ where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable $\theta = (T - T_\infty)/(T_w - T_\infty)$ and using von Karman transformations, equation (10) takes the form;

$$\frac{1}{\Pr}\frac{d^2\theta}{d\zeta^2} - H\frac{d\theta}{d\zeta} = 0, \quad (11)$$

Where $\Pr$ is the Prandtl number, $\Pr = c_p \mu / k$. The boundary conditions in terms of θ are expressed as

$$\theta(0) = 1, \qquad \theta(\infty) = 0, \quad (12)$$

Results and discussion:

The system of non-linear ordinary differential equations (5)–(7) and (11) is solved under the conditions given by equations (9) and (12) for the three components of the flow velocity and temperature distribution, using RK4 method with shooting technique. The resulting system of difference equations has to be solved in the infinite domain 0 < $\zeta < \infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance.

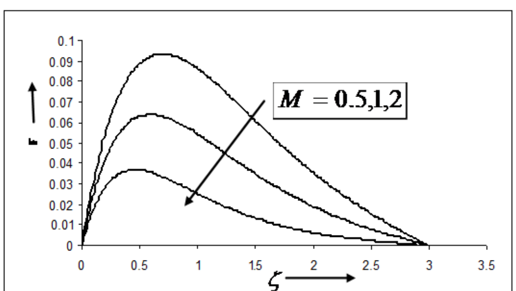


Fig. 1. Effect of M on the profile of F.

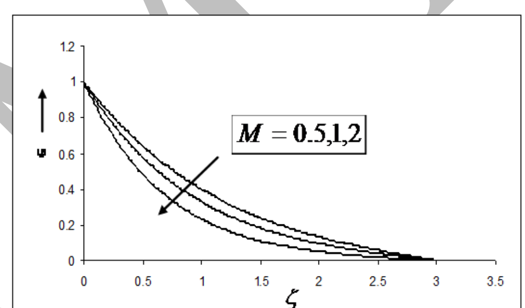


Fig. 2. Effect of M on the profile of G.

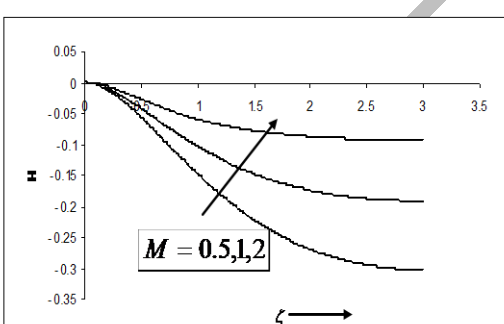


Fig. 3. Effect of M on the profile of H.

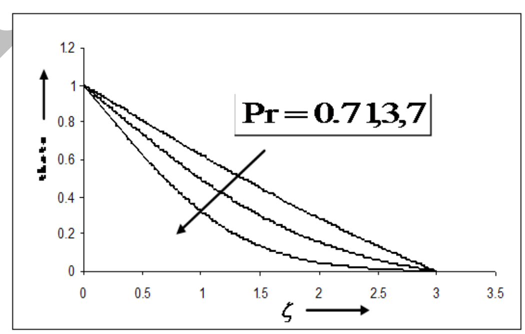


Fig. 4. Effect of $\Pr$ on the profile of θ.

Fig. 5. Effect of N on the profile of F.

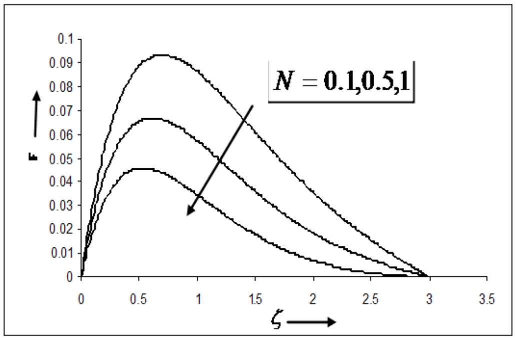


Fig. 6. Effect of N on the profile of H.

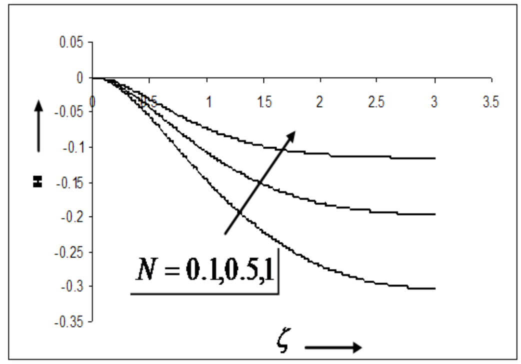


Fig. 7. Effect of ϕ on the profile of F.

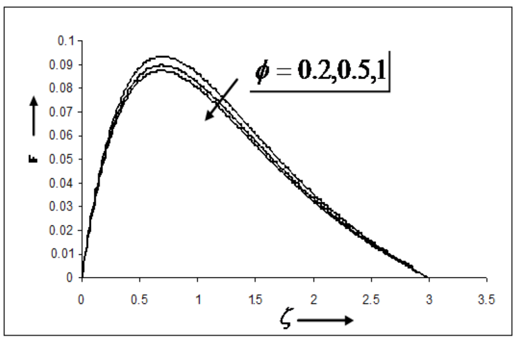
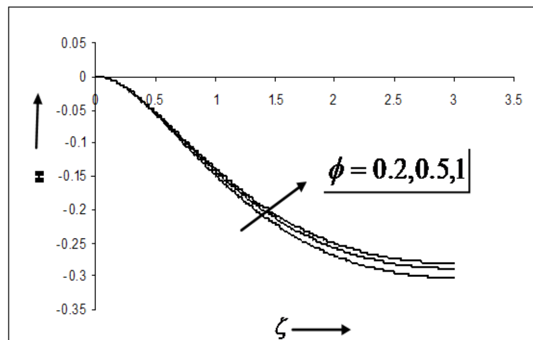
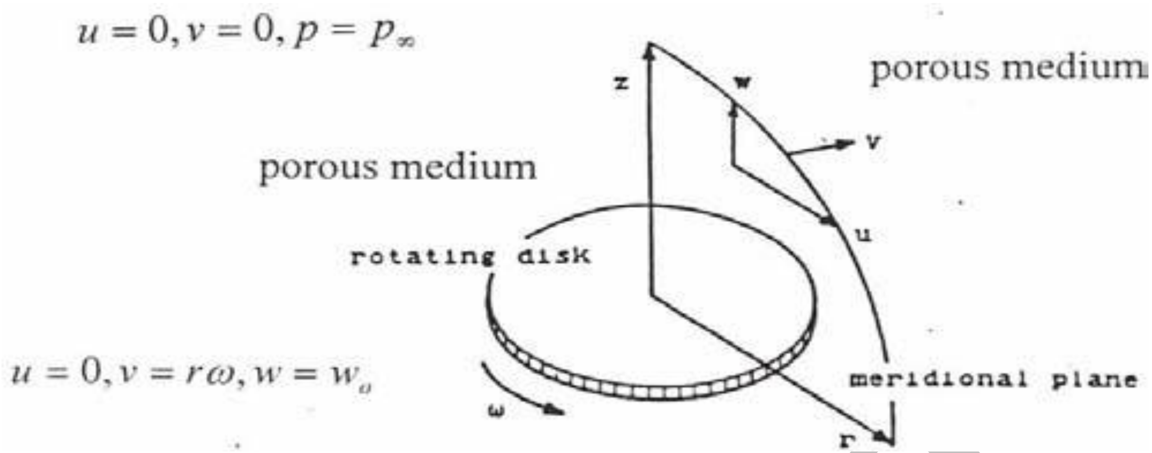


Fig. 8. Effect of ϕ on the profile of H.



Figures 1, 2, 3 present the steady state velocity components F, G and H, respectively, for various values of porosity parameter M ($M = 0.5, 1, 2$). In these figures, $\Pr = 0.71, N = 0.1$ and $\phi = 0.2$. It is clear from Figures 1, 2, 3 that increasing M decreases F, G and H for all ζ. Figure 4 indicate that, for $M = 0.5, N = 0.1$ and $\phi = 0.2$, increasing the Prandtl number ($\Pr = 0.71, 3, 7$) decreases temperature θ. Figures 5 and 6 show also the damping effect of the magnetic field which result in a reduction in the velocity components F and H for all ζ. Figures 7 and 8 present the steady state radial and axial velocity profiles F and H for various values of Hall parameter ($\phi = 0.2, 0.5, 1$) and for $\Pr = 0.71, \mathrm{N} = 0.1$ and $M = 0.5$. Figures 7 and 8 show that, increasing ϕ decreases F and H for all ζ.



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