

Reliability Analysis of a Fuzzy Pharmaceutical Plant

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Abstract

A pharmaceutical plant may be considered to consist of many subsystem in non-series parallel network i.e. bridge network. The study of fuzzy analysis can help in increasing the quality and production of the plant. The mathematical model has been developed for the plant with the help of Markov Process. In the present communication, first the available materials are outlined followed by the methods for calculating the capacities of a pharmaceutical system and for comparing the results are described.

Keywords: Fuzzy sets, Fuzzy numbers, Defuzzification, Markov-Process, Differential-difference Equations

Introduction

The pharmaceutical industry is an important industrial sector, both in terms of providing vital medicines for the well-being of people as well as being an employer. In the 27 member states of the European Union the pharmaceutical industry directly employs over 500,000 people in about 4,400 companies manufacturing various kinds of pharmaceutical preparations including vaccines, homeopathic preparations, dental fillings and bandages (Anon, 2010, 2011). Among these pharmaceutical preparations, parenterals (i.e. injectable drugs) form a large and important group. Many vaccines against ailments such as tetanus, rubella, measles, flu, mumps and hepatitis must be delivered to the body as injectables. The importance of parenterals is further illustrated by a list of 306 essential drugs compiled by World Health Organisation (WHO) (Anon, 2011), of which about 44% are injectables (Anon, 2011) requiring Water for Injection (WFI) as a delivery vehicle. In the future injectable drugs may become even more prevalent as the vast majority of biopharmaceuticals in development or in production must be injected into the bloodstream (Walsh, 2006, Ronald, 2008). This is because the molecules produced by biotechnology i.e. proteins, hormones and antibodies are generally speaking large molecules (typically 100 to 1,000 times larger than drugs based on chemical synthesis), which often cannot be delivered to the intended target via the oral route. In contrast the small molecules based on chemical synthesis typically allow oral route delivery (Walsh, 2006, Ronald, 2008).

Zadeh(1978) suggested a paradigm shift from the theory of total denial and affirmation to a theory of grading, to give new concept of sets called fuzzy sets. Fuzzy sets can express the gradual transition of the system from working state to failed state. The crisp set theory only dichotomizes the system in working state and failed state but fuzzy state theory can cover up all possible states between a fully working state and completely failed state. This approach to the reliability theory is known as profust reliability wherein the binary state assumption is replaced by Fuzzy state assumption.

The objective of the present research paper is to study the fuzzy analysis of Pharmaceutical plant. A Pharmaceutical plant may consists of many subsystems namely as Compression machine, Weighing machine, Sifter machine, Mass Mixer, Granulator, Fluid Bed Dryer, Blender, Rotary machine, Coating machine etc. These units are arranged in non-series parallel i.e. bridge network. Failure and repair rates of each subsystem are assumed to be fuzzy. The mathematical model of the plant has been developed using Markov Process. The differential equations have been developed with the help of probabilistic approach with fuzzy failure rates and repair rates. Equations are being solved with the help of Laplace Transform. Observations of the present research paper can be considered useful for the fuzzy analysis of the Pharmaceutical plat considered.

FUZZY SETS: - A set is a well-defined collection of objects. It has a sharp boundary to distinguish which element of the universe of discourse belongs to the set. In real life applications situations, it is not possible to distinct these elements by such a sharp layer due to the uncertainty involved. A fuzzy set is a set that consists of the elements having varying degrees of belongingness in the sets. So these situations may be better explained by the fuzzy sets, the set which contains all the elements of the universe but with different degrees of membership. A crisp set A may be defined over a universe X may be characterized by its characteristic function χ_A as $A = \{(x, \chi_A(x))\}$

Where $\chi_A : X \rightarrow \{0,1\}$ is defined by

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

In analogy to the characteristic function, we can define a function called membership function $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ to characterize a fuzzy set defined over the universe X as follows

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i)) : x_i \in X\}$$

Where, $\mu_{\tilde{A}}(x) \in [0, 1]$

DEFUZZIFICATION: - In certain situations one needs a crisp output when the input number is fuzzy. Defuzzification is the tool that makes it possible. We have several methods of defuzzification in the literature. Here in this work we have used the centroid method, which is given by the following expression.

$$x^* = \frac{\int \mu_{\tilde{A}}(x) \cdot x \, dx}{\int \mu_{\tilde{A}}(x) \cdot dx}$$

Where \tilde{A} is a fuzzy set, which is the union of two or more fuzzy sets i.e. $\tilde{A} = \tilde{A}_1 \cup \tilde{A}_2 \cup \dots \cup \tilde{A}_n$ and \cup is a standard fuzzy union.

FUZZY NUMBERS: -

Fuzzy Numbers: A fuzzy set A defined on R must possess the following properties to qualify as a fuzzy number.

- $\mu_A(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$
- $\mu_A(x)$ is strictly increasing on $[c, a]$ and strictly decreasing on $[b, d]$
- $\mu_A(x) = 1$ for all $x \in [a, b]$

Particularly it may be, $c = -\infty$ or $a=b$ or $b=d$ or $d = \infty$. $\mu_A(x)$ is a membership function for A.

Trapezoidal fuzzy numbers: - If the membership function of a fuzzy number is expressed as below.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a_1 \text{ or } x \geq a_3 \\ \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \leq x \leq a_4 \end{cases}$$

$$x, a_1, a_2, a_3, a_4 \in R$$

Assumptions and Notations: -

- (i) Initially all units of the system are in perfect condition.
- (ii) When failure comes to some module the system immediately takes reconfiguration Operation, with negligible time.
- (iii) One repairman is available to repair one faulty module at any time, except when the system is fully failed.
- (iv) Functioning of repaired unit (reconfiguration) is performed successfully.
- (v) Repair rate is constant from state S_1 to S_2 or from S_2 to S_3 .

- S_i : represents the system state that I active module is available
($i = 0, 1, 2, \dots$).
- $P_i(t)$: represents the probability that the system remains in state S_i at time t.
($i = 0, 1, 2, \dots$).
- C : represents the system coverage factor.
- $\mu_s(s_i)$: Fuzzy success state S with membership function. ($i = 0, 1, 2, \dots$).
- $\mu_f(s_i)$: Fuzzy failure state F with membership function. ($i = 0, 1, 2, \dots$).
- $\tilde{\lambda}$: Fuzzy failure rate.
- μ : Constant repair rate

MATHEMATICAL FORMULATION AND SOLUTION OF THE MODEL: -

The differential equations of the system are: -

$$P'_i(t) + 3\lambda p_i = \mu p_1$$

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$$P'_2(t) + 2\lambda p_2 = \mu p_1$$

$$P_1'(t) + (\lambda + \mu)p_1 = 2\lambda cp_2$$

$$P_0'(t) = \lambda(1-c)p_1 + 2\lambda(1-c)p_2 + \dots + \dots + 3\lambda(1-c)p_i$$

$$p_i(0) = 1$$

SOLUTION OF THE MODEL: -

$$P_i(t) = c^{3-i} e^{-i\lambda t} \left(\frac{3}{i}\right) (1 - e^{-\lambda t})^{3-i}$$

$$i = 0, 1, 2, 3, \dots, n$$

$$P_0(t) = 1 - \sum_{i=1}^n P_i(t)$$

Where, $\binom{n}{i} = \frac{n!}{i!(n-i)!}$

Now, we define one fuzzy success state S and one fuzzy failure state F with membership function,

$$\mu_s(S_i) = \frac{i}{n}; i = 0, 1, 2, \dots$$

$$\mu_f(S_i) = \frac{n-i}{n}; i = 0, 1, 2, \dots$$

Then, we have a transition from success to failure states,

$$\mu_{TSF}(m_{ij}) = \begin{cases} i - \frac{j}{n} & \text{if } i > j \\ 0 & \text{if } i \leq j \end{cases}$$

We have $p_3(0) = 1$, so the system profust reliability become,

$$R(t) = \sum_{j=1}^3 \frac{j}{n} P_j(t)$$

$$= \sum_{j=1}^3 \frac{j}{n} c^{n-j} c^{-j\lambda t} \frac{n!}{j!(n-j)!} (1 - e^{-\lambda t})^{n-j}$$

The system profust availability is also,

$$A(t) = \sum_{j=1}^n \frac{j}{n} P_j(t)$$

If the system success / failure is defined clearly i.e.

$$\mu_S(S_i) = \begin{cases} 1; i \geq k \\ 0; i \leq k \end{cases}$$

$$\mu_F(S_i) = \begin{cases} 0; i \geq k \\ 1; i \leq k \end{cases}$$

Where k is some positive integer, then

$$R(t) = \sum_{i=k}^n P_i(t)$$

i.e. the system profust reliability reduces to the system probist reliability.

Fuzzy Reliability analysis:-

Equations are solved with the help of c-language program with the parameters given below.

($\lambda = .00112$ / hour, $t=500$ hours, $c=0.8$)

Time X(hours)	0	50	100	150	200	250	300	350	400	450	500
R(t)	1	0.9438	0.9177	0.87458	0.8475	0.8168	0.7877	0.7498	0.6985	0.6748	0.6412
Time X(hours)	550	600	650	700	750	800	850	900	8500	1000	
R(t)	0.6128	0.5957	0.5876	0.5743	0.5585	0.5479	0.5397	0.5285	0.5212	0.5197	

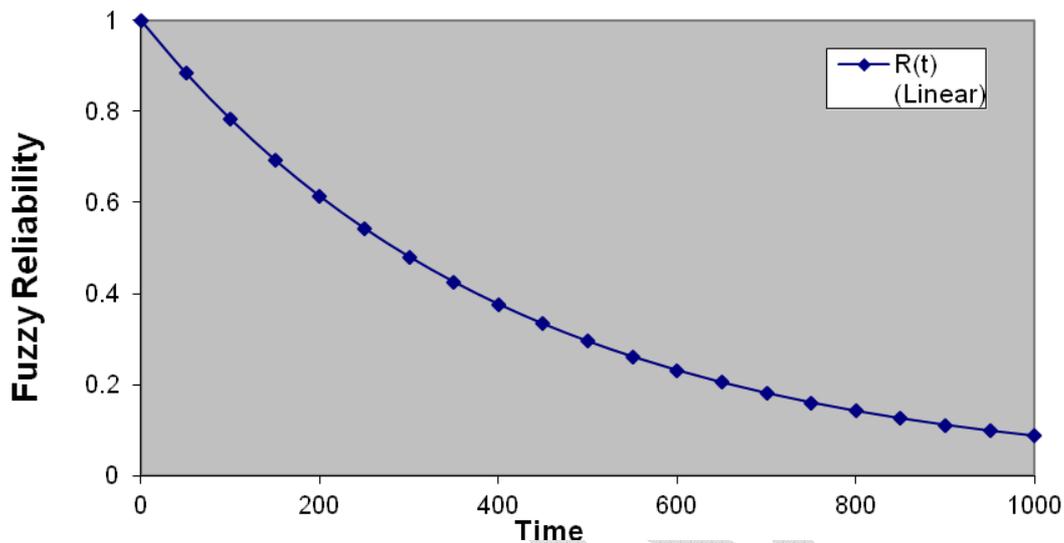


Fig. 1 Curve indicating fuzzy reliability vs. time

Conclusion and Result discussion:-

In this present research paper fuzzy reliability computer program has been described. There are various advantages to use this method than others because

- (i) Differential – Difference equations are easy to solve with the help of Laplace Transform.
- (ii) This methodology is very close to reality and the results are clear to real life.
- (iii) Results are more accurate and very precise.
- (iv) Failure and repair rates with membership function are very clear.

The methodology of the present research paper can increase the quality and production of the pharmaceutical plant. The proposed method can be applied to complex systems with many sub states. This includes a large system of differential-difference equations. Using this method one can easily study the fuzzy analysis of reliability of the plant with respect to time. Figure 1 shows the variation of reliability with respect to time. Initially fuzzy reliability decreases gradually with respect to time and become almost stable after long duration of time.

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